

University of Kalyani

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS



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No.....

Date : 21/04/2017

**SEMESTER SYSTEM COURSE STRUCTURE
(CHOICE BASED CREDIT)**

**M. SC. COURSE IN MATHEMATICS
(PURE AND APPLIED STREAMS)**



DEPARTMENT OF MATHEMATICS

FACULTY OF SCIENCE

UNIVERSITY OF KALYANI

M. Sc. Course in Mathematics

(Pure and Applied Streams)

Total Marks : 1600

(Total four semester course carrying 400 marks in each semester)

Outline of the Choice Based Credit Semester System

Transaction Categories: C: Common to both Streams; A: Applied Stream; P: Pure Stream; CB:

Choice Based Course; O: Optional Subject; PW: Project Work

Evaluation Categories: SEE: Semester End Examination; IA: Internal Assessment

Course	Topics	Marks		Credit	Hrs./ Wk
		SEE	IA		
SEMESTER I					
MATC 1.1	Real Analysis – I ; Complex Analysis – I ; Functional Analysis – I	80	20	4	8
MATC 1.2	Ordinary Differential Equations ; Partial Differential Equations	80	20	4	8
MATC 1.3	Mechanics – I ; Abstract Algebra – I ; Operations Research–I	80	20	4	8
MATC 1.1 TO MATC 1.3 ARE COMMON TO BOTH PURE AND APPLIED STREAMS					
MATA 1.4	Mechanics of Solids ; Non-linear Dynamics	80	20	4	8
MATP 1.4	Differential Geometry–I ; Topology–I	80	20	4	8
SEMESTER II					
MATCB 2.1	History of Mathematics ; Operations Research ; Linear Algebra ; Dynamical Systems	80	20	4	4
MATCB 2.1 IS BASED ON THE CHOICES OF THE STUDENTS OF OTHER DEPARTMENT(S)					
MATC 2.2	Real Analysis – II ; Complex Analysis – II ; Functional Analysis – II	80	20	4	8
MATC 2.3	Mechanics – II ; Abstract Algebra– II ; Operations Research – II	80	20	4	8
MATC 2.2 TO MATC 2.3 ARE COMMON TO BOTH PURE AND APPLIED STREAMS					
MATA 2.4	Mechanics of Fluids ; Stochastic Process	80	20	4	8
MATP 2.4	Differential Geometry–II ; Topology–II	80	20	4	8
SEMESTER III					
MATC 3.1	Linear Algebra ; Special Functions ; Integral Equations & Integral Transforms	80	20	4	8
MATC 3.1 TO MATC 3.2 ARE COMMON TO BOTH PURE AND APPLIED STREAMS					

Course	Topics	Marks		Credit	Hrs./ Wk
		SEE	IA		
SEMESTER III (Contd..)					
MATA 3.2	Fuzzy Set Theory ; Computer Programming in 'C' (Theory); Numerical Analysis (Practical)	80	20	4	8
MATA 3.3	Dynamical System ; Numerical Analysis (Theory)	80	20	4	8
MATA 3.4	Mathematical Biology ; Electro Magnetic Theory	80	20	4	8
MATP 3.2	Fuzzy Set Theory ; Computer Programming in 'C' (Theory); Computer Programming in 'C' (Practical)	80	20	4	8
MATP 3.3	Topological Groups ; Measure Theory	80	20	4	8
MATP 3.4	Calculus of R^n + Operator Theory	80	20	4	8
SEMESTER IV					
MATC 4.1	Discrete Mathematics ; Probability and Statistical Methods	80	20	4	8
MATC 4.1 IS COMMON TO BOTH PURE AND APPLIED STREAMS					
MATO 4.2	Optional Paper – I	80	20	4	6
MATO 4.3	Optional Paper – II	80	20	4	6
The Optional Papers be offered to the students on the basis of availability of Teachers and within the Framed Syllabi of the Optional Papers					
MATPW 4.4	Project Work	Disserta tion	Presen tation	Viva Voce	Credit
		50	30	20	4
Each student has to undergo through a Project Work under the guidance of the teacher(s) of the Department, and on the basis of subject interest of the students in advanced field of study in different areas of Mathematics.					

Semester-Wise Distribution of Subjects with Marks

I. First Semester:

<u>Course Name</u>	<u>Subject-Wise Marks</u>	<u>Total Marks</u>
MATC 1.1: Real Analysis – I + Complex Analysis – I + Functional Analysis – I		100
	(SEE: 25+30+25; IA: 7+6+7)	
MATC 1.2: Ordinary Differential Equations + Partial Differential Equations		100
	(SEE: 40+40; IA: 10+10)	
MATC 1.3: Mechanics – I (Potential Theory) + Abstract Algebra – I		100
+ Operations Research–I	(SEE: 30+25+25; IA: 6+7+7)	
<u>*MATC 1.1 – MATC 1.3 are common to both the pure and applied streams.</u>		
MATA 1.4: Mechanics of Solids + Non-linear Dynamics		100
(Applied Stream)	(SEE: 40+40; IA: 10+10)	
MATP 1.4: Differential Geometry–I + Topology–I		100
(Pure Stream)	(SEE: 40+40; IA: 10+10)	

II. Second Semester :

<u>Course Name</u>	<u>Subject-Wise Marks</u>	<u>Total Marks</u>
MATCB 2.1: History of Mathematics + Operations Research		100
+ Linear Algebra + Dynamical Systems	(SEE: 20+20+20+20; IA: 5+5+5+5)	
<u>MATCB 2.1 is based on the Choices of the Students of other Department(s)</u>		
MATC 2.2: Real Analysis – II + Complex Analysis – II+ Functional Analysis – II		100
	(SEE: 25+25+30; IA: 7+7+6)	
MATC 2.3: Mechanics– II (Classical Mechanics) + Abstract Algebra– II		100
+ Operations Research – II	(SEE: 25+25+30; IA: 7+7+6)	
<u>*MATC 2.2 – MATC 2.3 are common to Both the Pure and Applied Streams Students.</u>		
MATA 2.4: Mechanics of Fluids + Stochastic Process		100
(Applied Stream)	(SEE: 50+30; IA: 10+10)	
MATP 2.4: Differential Geometry–II + Topology–II		100
(Pure Stream)	(SEE: 40+40; IA: 10+10)	

III. Third Semester :

<u>Course Name</u>	<u>Subject</u>	<u>Total Marks</u>
MATC 3.1:	Linear Algebra + Special Functions + Integral Equations & Integral Transforms	100 (SEE: 30+20+30; IA: 10+5+5)

***MATC 3.1 is Compulsory to Both the Pure and Applied Streams Students.**

APPLIED STREAM

<u>Course Name</u>	<u>Subject</u>	<u>Total Marks</u>
MATA 3.2:	Fuzzy Set Theory + Computer Programming in 'C' (Theory) + Numerical Analysis (Practical)	100 (SEE: 20+30+30; IA: 6+7+7)
MATA 3.3:	Dynamical System + Numerical Analysis (Theory)	100 (SEE: 40+40; IA: 10+10)
MATA 3.4:	Mathematical Biology + Electro Magnetic Theory	100 (SEE: 55+25; IA: 15+5)

PURE STREAM

MATP 3.2:	Fuzzy Set Theory + Computer Programming in 'C' (Theory) + Computer Programming in 'C' (Practical)	100 (SEE: 20+30+30; IA: 6+7+7)
MATP 3.3:	Topological Groups+ Measure Theory	100 (SEE: 40+40; IA: 10+10)
MATP 3.4:	Calculus of R^n + Operator Theory	100 (SEE: 40+40; IA: 10+10)

IV. Fourth Semester :

<u>Course Name</u>	<u>Subject</u>	<u>Total Marks</u>
MATC 4.1:	Discrete Mathematics + Probability and Statistical Methods	100 (SEE: 50+30; IA: 10+10)

***MATC 4.1 is common to Both the Pure and Applied Streams Students.**

***MATO 4.2 & MATO 4.3: Two Separate Optional Subjects with 100 Marks (SEE : 80; IA: 20) under each Course as Special Fields of Study.**

(The Optional Subjects are listed in a Separate Sheet)

- The Optional Subjects be offered to the Students of both the Streams (Pure and Applied) on the basis of availability of Teachers as Resource Persons and within the Framed Syllabi of the Optional Subjects.

<u>Course Name</u>	<u>Subject</u>	<u>Total Marks</u>
MATPW 4.4:	Project Work	100
(Dissertation: 50 + Seminar presentation: 30 + Viva-voce: 20)		

***MATPW 4.4 is Compulsory to Both the Pure and Applied Streams Students.**

Examination Related Course Criteria (Unit 16) :

- Project Work be made by the students under the guidance of the teacher(s) of the Department, and on the basis of subject interest of the students in advanced field of study in different areas of **Mathematics**.
 - Dissertation of the Project Work be prepared by individual student and the same be submitted to the HOD after countersigned by the concerned teacher(s) and prior to commencement of Viva-Voce.
 - Project Work related Record** be maintained by the Department.
 - Seminar presentation and Viva–Voce Examination be conducted by the Department.
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The List of Optional Papers

Applied Stream	Pure Stream
#1. Advanced Operations Research – I	#1. Advanced Operations Research – I
#2. Advanced Operations Research – II	#2. Advanced Operations Research – II
#3. Fuzzy Sets and Systems	#3. Fuzzy Sets and Systems
4. Advanced Solid Mechanics	4. Advanced Real Analysis
5. Advanced Fluid Mechanics	5. Advanced Partial Differential Equations
6. Computational Fluid Mechanics	6. Advanced Complex Analysis – I
7. Magneto-Fluid Mechanics	7. Advanced Complex Analysis – II
8. Plasma Physics	8. Advanced Functional Analysis
9. Mathematics of Finance & Insurance	9. Set-Valued Analysis
10. Seismology	10. Abstract Harmonic Analysis
11. Computational Biology	11. Advanced General Topology
12. Mathematical Biology	12. Advanced Algebraic Topology
13. Dynamical Oceanography	13. Advanced Algebra – I
14. Applied Functional Analysis	14. Advanced Algebra – II
15. Advanced Numerical Analysis (Theory and Practical)	15. Advanced Geometry – I
16. Compressible Fluid Dynamics	16. Advanced Geometry – II
	17. Ergodic Theory and Topological Dynamics

#Syllabi for Advanced Operations Research – I, Advanced Operations Research – II and Fuzzy Sets and Systems are common to both the Pure and Applied Streams

I. The First Semester

MATC 1.1

Real Analysis – I

(Pure and Applied Streams)

Marks : 32 (SEE: 25; IA: 07)

Cardinal number : Definition, Schröder-Berstein theorem, Order relation of cardinal numbers, Arithmetic of cardinal numbers, Continuum hypothesis (4)

Cantor's set : Construction and its presentation as an uncountable set of measure zero (2)

Functions of bounded variation : Definition and basic properties, Lipschitz condition, Jordan decomposition, Nature of points of discontinuity, Nature of points of non-differentiability, Convergence in variation (Helly's First theorem). (5)

Absolutely continuous functions : Definition and basic properties, Deduction of the class of all absolutely continuous functions as a proper subclass of all functions of bounded variation, Characterization of an absolutely continuous function in terms of its derivative vanishing almost everywhere. (5)

Riemann-Stieltjes integral : Existence and basic properties, Integration by parts, Integration of a continuous function with respect to a step function, Convergence theorems in respect of integrand, convergence theorem in respect of integrator (Helly's Second theorem). (5)

Gauge partition : Definition of a delta-fine tagged partition and its existence, Lebesgue's criterion for Riemann integrability, Delta-fine free tagged partition and an equivalent definition of the Riemann integral. (4)

References :

1. W. Rudin : Principles of Mathematical Analysis.
2. D. V. Widder : Laplace Transform.
3. H. L. Royden : Real Analysis.
4. B. K. Lahiri and K. C. Ray : Real Analysis.
5. A. G. Das : The Generalized Riemann Integral.
6. A. G. Das : Theory of Integration – The Riemann, Lebesgue and Henstock-Kurzweil Integrals.
7. W. Sierpinsky : Cardinal Number and Ordinal Number.
8. I. P. Natanson : Theory of Integrals of a Real Variable (Vol. I and II).

Complex Analysis – I
(Pure and Applied Streams)
Marks : 36 (SEE: 30; IA: 06)

Riemann's sphere, point at infinity and the extended complex plane. (2)

Functions of a complex variable, limit and continuity. Analytic functions, Cauchy-Riemann equations. Complex integration. Cauchy's fundamental theorem (statement only) and its consequences. Cauchy's integral formula. Derivative of an analytic function, Morera's theorem, Cauchy's inequality, Liouville's theorem, Fundamental theorem of classical algebra. (20)

Uniformly convergent series of analytic functions. Power series. Taylor's theorem. Laurent's theorem. (8)

References :

1. A. I. Markushevich : Theory of Functions of a Complex Variable (Vol. I, II and III).
2. R. V. Churchill and J. W. Brown : Complex Variables and Applications.
3. E. C. Titchmarsh : The Theory of Functions.
4. E. T. Copson : An Introduction to the Theory of Functions of a Complex Variable.
5. J. B. Conway : Functions of One Complex Variable.
6. L. V. Ahlfors : Complex Analysis.
7. H. S. Kasana : Complex Variables – Theory and Applications.
8. S. Narayan and P. K. Mittal : Theory of Functions of a Complex Variable.
9. A. K. Mukhopadhyay : Functions of Complex Variables and Conformal Transformation.
10. J. M. Howie : Complex Analysis.

Functional Analysis–I
(Pure and Applied Streams)
Marks : 32 (SEE: 25; IA: 07)

Metric spaces. Brief discussions of continuity, completeness, compactness. Hölder's and Minkowski's inequalities (statement only). (5)

Baire's (category) theorem. The spaces $\mathbb{R}^k, \mathbb{C}^k, C[a, b]$ and ℓ_p . Banach's fixed point theorem, applications to solutions of certain systems of linear algebraic equations, Fredholm's integral equation of the second kind, implicit function theorem. Kannan's fixed point theorem. (8)

Real and Complex linear spaces. Normed induced metric. Banach spaces, Riesz's lemma. Finite dimensional normed linear spaces and subspaces, completeness, compactness criterion, equivalent norms. (12)

References :

1. E. Kreyszig : Introductory Functional Analysis with Applications.
2. G. Bachman and L. Narici : Functional Analysis.
3. W. Rudin : Functional Analysis.
4. N. Dunford and L. Schwartz : Linear Operators (Part I).
5. A. E. Taylor : Introduction to Functional Analysis.
6. B. V. Limaye : Functional Analysis.
7. K. Yoshida : Functional Analysis.
8. B. K. Lahiri : Elements of Functional Analysis.

MATC 1.2

Ordinary Differential Equations

(Pure and Applied Streams)

Marks : 50 (SEE: 40; IA: 10)

Existence of solutions : Picard's Existence theorem for equation $dy / dx = f(x,y)$, Gronwall's lemma, Picard-Lindelöf method of successive approximations. (8)

Solutions of linear differential equations of nth order. Wronskian, Abel's identity, linear dependence and independence of the solution set, Fundamental set of solutions. Green's function for boundary value problem and solution of non-homogenous linear equations. Adjoint and self-adjoint equations. Lagrange's identity. Sturm's separation and comparison theorems for second order linear equations. Regular Sturm-Liouville problems for second order linear equations. Eigen values and eigen functions, expansion in eigen functions. (20)

Solution of linear ordinary differential equations of second order in complex domain. Existence of solutions near an ordinary point and a regular singular point, Solutions of Hyper geometric equation and Hermite equation, Introduction to special functions (12)

References :

1. M. Birkhoff and G. C. Rota : Ordinary Differential equations.

2. E. L. Ince : Ordinary Differential Equations.
3. G. F. Simmons : Differential Equations.
4. E. E. Coddington and N. Levinson : Theory of Ordinary Differential Equations.
5. E. T. Copson : An Introduction to the Theory of Functions of a Complex Variable.

Partial Differential Equations

(Pure and Applied Streams)

Marks : 50 (SEE: 40; IA: 10)

Introduction and pre-requisite, Genesis and types of solutions of Partial Differential Equations, First order Partial Differential Equations, Classifications of First Order Partial Differential Equations, Charpit's Method for the solution of First Order non-linear Partial Differential Equation. (6)

Linear Partial Differential Equations of second and higher order, Linear Partial Differential Equation with constant coefficient, Solution of homogeneous irreducible Partial Differential Equations, Method of separation of variables, Particular integral for irreducible non-homogeneous equations, Linear Partial Differentiable equation with variable coefficients, Canonical forms, Classification of Second order Partial Differential Equations, Canonical transformation of linear Second order Partial Differential equations, (7)

Parabolic equation, Initial and boundary conditions, Heat equation under Dirichlet's Condition, Solution of Heat equation under Dirichlet's Condition, Solution of Heat equation under Neuman Condition, Solution of Parabolic equation under non-homogeneous boundary condition. (9)

Hyperbolic equation, occurrence of wave equations, in Mathematical Physics, Initial and boundary conditions, Initial value problem, D' Alembert's solutions, vibration of a string of finite length, Initial value problem for a non-homogeneous wave equation. (9)

Elliptic equations, Gauss Divergence Theorem, Green's identities, Harmonic functions, Laplace equation in cylindrical and spherical polar coordinates, Dirichlet's Problem, Neumann Problem, (9)

References :

1. I. N. Sneddon : Elements of Partial Differential Equations.
2. E. Epstein : Partial Differential Equations.
3. G. Greenspan : Introduction to Partial Differential Equations.
4. M. G. Smith : Introduction to the Theory of Partial Differential Equations.

MATC 1.3

Mechanics – I (Potential Theory)

(Pure and Applied Streams)

Marks : 36 (SEE: 30; IA: 06)

Concept of potential and attraction for line, surface and volume distributions of matter. Laplace's equation, problems of attraction and potential for simple distribution of matter. (5)

Existence and continuity of first and second derivatives of potential within matter. Poisson's equation, work done by mutual attraction, problems. (7)

Integral theorem of potential theory (statement only) Green's identities, Gauss' average value theorem, continuity of potential and discontinuity of normal derivative of potential for a surface distribution, potential for a single and double layer, Discontinuity of potential. (8)

Boundary value problems of potential theory. Green's function, solution of Dirichlet's problem for a half-space. (5)

Solid and surface spherical harmonics. (5)

References :

1. O. D. Kellog : Theory of Potential.
2. P. K. Ghosh : Theory of Potential.
3. A. S Ramsey : Newtonian Attraction.
4. T. M. MacRobert : Spherical Harmonics.

Abstract Algebra –I

(Pure and Applied Streams)

Marks : 32 (SEE: 25; IA: 07)

Preliminaries: Review of earlier related concepts-Groups and their simple properties. (3)

Class equations on groups and related theories: Conjugacy class equations, Cauchy's theorem, p -Groups, Sylow theorems and their applications, simple groups. (6)

Direct Product on groups: Definitions, discussion on detailed theories with applications. (6)

Solvable groups: Related definitions and characterization theorems, examples. (6)

Group action: Definition and relevant theories with applications. (4)

References :

1. I. N. Herstein – Topics in Algebra.
2. Malik, Mordeson and Sen – Fundamentals of Abstract Algebra.
3. M. R. Adhikari and Abhishek Adhikari – Groups, Rings and Modules with Applications.
4. S. Lang – Algebra.
5. J. B. Fraleigh – A First Course in Abstract Algebra.
6. N. Jacobson – Lecturers in Abstract Algebra.
7. B. C. Chatterjee – Abstract Algebra (Vol. 1).
8. J. A. Gallian – Contemporary Abstract Algebra.
9. T. W. Hungerford – Algebra.
10. Luthar & Passi – Algebra (Vol. 1).

Operations Research–I
(Pure and Applied Streams)
Marks : 32 (SEE: 25; IA: 07)

Extension of Linear Programming Methods : Theory of Revised Simplex Method and algorithmic solution approaches to linear programs, Dual-Simplex Method, Decomposition principle and its use to linear programs for decentralized planning problems. (8)

Integer Programming (IP) : The concept of cutting plane for linear integer programs, Gomory's cutting plane method, Gomory's All-Integer Programming Method, Branch-and-Bound Algorithm for general integer programs. (6)

Sequencing Models : The mathematical aspects of Job sequencing and processing problems, Processing n jobs through Two machines, processing n jobs through m machines. (5)

Nonlinear Programming (NLP) : Convex analysis, Necessary and Sufficient optimality conditions, Cauchy's Steepest descent method, Karush-Kuhn-Tucker (KKT) theory of NLP, Wolfe's and Beale's approaches to Quadratic Programs. (6)

References :

1. Linear Programming – G. Hadley.
2. Mathematical Programming Techniques – N. S. Kambo.
3. Nonlinear and Dynamic Programming – G. Hadley.
4. Operations Research – K. Swarup, P. K. Gupta and Man Mohan.

5. Operations Research – H. A. Taha.
6. Operations Research – S. D. Sharma.
7. Introduction to Operations Research – A. Frederick, F. S. Hillier and G. J. Lieberman.
8. Optimization : Theory and Applications – S. S. Rao.
9. Nonlinear and Mixed-Integer Optimization – Christodoulos A. Floudas.

*** MATC 1.1 – MATC 1.3 are common to both the pure and applied streams**

APPLIED STREAM

MATA 1.4

Mechanics of Solids

(Applied Stream)

Marks : 50 (SEE: 40; IA: 10)

Brief discussion of tensor transformation, symmetric tensor, alternating tensor. Analysis of strain, Normal strain, shearing strain and their geometrical interpretations. Strain quadratic of Cauchy, Principal strains, Invariants, Saint-Venant's equations of compatibility, equivalence of Eulerian and Lagrangian components of strain in infinitesimal deformation. (9)

Analysis of stress, stress tensor, Equations of equilibrium and motion. Stress quadric of Cauchy. Principal stress and invariants, strain energy function. (7)

Graphical representation of elastic deformation. Equations of elasticity. Generalized Hooke's law. Homogeneous isotropic media. Elastic moduli for isotropic media. Equilibrium and dynamical equations for an isotropic elastic solid. connections of the strain energy function with Hooke's Law, uniqueness of solutions. Clapeyron's Theorem, Beltrami-Michell compatibility equations, Saint-Venant's principle. (8)

Equilibrium of isotropic elastic solid : Deformations under uniform pressure. Deformations of prismatical bar stretched by its own weight and a cylinder immersed in a fluid, twisting of circular bar by couples at the ends. (4)

Torsion : Torsion of cylindrical bars, Torsional rigidity, Torsion function, Lines of shearing stress, simple problems related to circle, ellipse and equilateral triangle. (4)

Two-dimensional problems : Plane strain, Plane stress, Generalised plane stress, Airy's stress function, General solution of biharmonic equation. Stresses and displacements in terms of complex potentials. Simple problems, stress function appropriate to problems of plane stress. (5)

Waves : Propagation of waves in an isotropic elastic medium, waves of dilatation and distortion. Plane waves. (3)

References :

1. S. Sokolnikoff – Mathematical Theory of Elasticity.
2. A. E. H. Love – A Treatise on the Mathematical Theory of Elasticity.
3. Y. C. Fung – Foundations of Solid Mechanics.
4. S. Timoshenko and N. Goodier – Theory of Elasticity.
5. R. V. Southwell – Theory of Elasticity.

Nonlinear Dynamics

(Applied Stream)

Marks: 50 (SEE: 40; IA: 10)

Linear autonomous systems: Linear autonomous systems, existence, uniqueness and continuity of solutions, diagonalization of linear systems, fundamental theorem of linear systems, the phase paths of linear autonomous plane systems, complex eigen values, multiple eigen values, similarity of matrices and Jordan canonical form, stability theorem, reduction of higher order ODE systems to first order ODE systems, linear systems with periodic coefficients. (15)

Linearization of dynamical systems: Two, three and higher dimension. Population growth. Lotka-Volterra system. (5)

Stability: Asymptotic stability (Hartman's theorem), Global stability (Liapunov's second method). Limit set, attractors, periodic orbits, limit cycles. Bendixon criterion, Dulac criterion, Poincare-Bendixon Theorem. (15)

Stability and bifurcation: Saddle-Node, transcritical and pitchfork bifurcations. Hopf-bifurcation. (5)

References :

1. D. W. Jordan and P. Smith (1998): Nonlinear Ordinary Equations- An Introduction to Dynamical Systems (Third Edition), Oxford Univ. Press.
2. L. Perko (1991): Differential Equations and Dynamical Systems, Springer Verlag.
3. F. Verhulst (1996): Nonlinear Differential Equations and Dynamical Systems, Springer Verlag.
4. H. I. Freedman - Deterministic Mathematical Models in Population Ecology.
5. Mark Kot (2001): Elements of Mathematical Ecology, Cambridge Univ. Press.

PURE STREAM

MATP 1.4

Differential Geometry – I

(Pure Stream)

Marks: 50 (SEE: 40; IA: 10)

Vector valued functions, Directional Derivatives, Total derivatives, Statement of Inverse and Implicit Function Theorems, Curvilinear coordinate system in E^3 . Reciprocal base system. Riemannian space. Reciprocal metric tensor, Christoffel symbols, Covariant differentiation of vectors and tensors of rank 1 and 2. (8)

Riemannian curvature tensor, Ricci tensor and scalar curvature. Space of constant curvature, Einstein space. On the meaning of covariant derivative. Intrinsic differentiation. Parallel vector field. (10)

Tensor Algebra on finite dimensional vector spaces, Inner product spaces, matrix representation of an inner product, , Linear functional, r-forms, Exterior product, Exterior derivative. (8)

Regular curves, curvature, torsion, curves in plane, signed curvature, curves in spaces, Serret Frenet formulae, Isoperimetric inequality, four vertex theorem. (7)

Introduction to surface, Definition example, first fundamental form of surfaces. (7)

References :

1. I. S. Sokolnikoff : Tensor Analysis, Theory and Applications to Geometry and Mechanics of Continua.
2. L. P. Eisenhart : An Introduction to Differential Geometry (with the use of Tensor Calculus).

3. T. Y. Thomas : Concepts from Tensor Analysis and Differential Geometry.
4. M. C. Chaki : A Text book of Tensor Calculus.
5. U. C. De, Absos Ali Shaikh and Joydeep Sengupta : Tensor Calculus.
6. U. C. De : Differential Geometry of Curves and Surfaces in E^3 (Tensor Approach).
7. Differential Geometry, Nirmala Prakasha
8. Linear algebra, K Hoffman, R. Kunze.
9. Differential Geometry of curves and surfaces, M. P Do Carmo
10. Calculus on Manifolds, M. Spivak

Topology–1

(Pure Stream)

Marks: 50 (SEE: 40; IA: 10)

Definition and examples of topological spaces. Basis for a given topology, necessary and sufficient condition for two bases to be equivalent, sub-base, topologizing of two sets from a sub base. Closed sets, closure and interior, their basic properties and their relations. Neighbourhoods, exterior and boundary, dense sets. Accumulation points and derived sets. Subspace topology.

Continuous, open, closed mappings, examples and counter examples, their different characterizations and basic properties, Pasting lemma, homeomorphism, topological properties.

The countability axioms, Separation axioms, Urysohn's lemma and Tietzes extension theorem (Statements only) and some of their applications.

References :

1. M. A. Armstrong, Basic Topology, Springer (India), 2004,
2. J.R. Munkres, Topology, 2nd Ed., PHI (India), 2002,
3. J. M. Lee : Introduction to topological Manifolds,
4. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw- Hill, New York, 1963.

II. The Second Semester

MATCB 2.1

Choice Based Paper

(Pure and Applied Streams)

Marks : 100

History of Mathematics. Objectives, Babylonian and Egyptian mathematics, Greek mathematics, Pythagoras, Euclid and the elements of geometry, Archimedes, Apollonius, Development of Trigonometry, Development of Algebra, Development of Analytic Geometry, Development of Calculus, Development of Selected Topics of Modern Mathematics, Modern geometries, Modern algebra, Methods of real analysis. (20)

Operations Research. Formulation of linear programming models. Graphical solution. (2)

Basic solution (BS) and Basic Feasible Solution (BFS), Degenerate and non-degenerate BFS, Convex set, convex hull, convex polyhedron, extreme points, hyper plane. Standard form of LPP.

Simplex method. Charnes' Big – M method. (8)

Transportation and assignment problems. (4)

Components of a network. Shortest Path Method: Dijkstra's Algorithm, Floyd's Algorithm. A brief introduction to PERT and CPM, Components of PERT/CPM Network and precedence relationships, Critical path analysis. (6)

Linear Algebra. Matrix: definition, order, symmetric and skew symmetric matrices, determinant of a matrix, elementary properties of determinants, inverse of a matrix, normal form of a matrix, rank of a matrix, elementary concept of a vector space, linear dependence and independence of vectors, basis of a vector space, row space, column space, solution of system of linear equations, Cramer's rule, Eigen values and Eigen vectors of matrices, Cayley Hamilton Theorem, Diagonalization of matrices. (20)

Dynamical Systems.

Linearization of dynamical systems: Two, three and higher dimension. Population growth. Lotka-Volterra system. (5)

Stability: Asymptotic stability (Hartman's theorem), Global stability (Liapunov's second method). Limit set, attractors, periodic orbits, limit cycles. Bendixon criterion, Dulac criterion, Poincare-Bendixon Theorem. Floquet's theorem. (10)

Stability and bifurcation: Routh-Hurwitz criterion for nonlinear systems. Saddle-Node, transcritical and pitchfork bifurcations. Hopf- bifurcation. (5)

References:

1. D.M. Burton, The History of Mathematics, Allyn and Bacon, 5th edition
2. Carl B. Boyer and Uta C. Merzbach, A History of Mathematics 3rd Edition .
3. Florian Cajori , A History of Mathematics (Paperback).
4. J.H. Eves, An Introduction to the History of Mathematics, Saunders, 1990.
5. Clifford A. Pickover , The Math Book: From Pythagoras to the 57th Dimension, 250 Milestones in the History of Mathematics (Sterling Milestones)Paperback –February 7, 2012.
6. Jacqueline Stedall, The History of Mathematics: A Very Short Introduction 1st Edition.
7. Dirk J. Struik , A Concise History of Mathematics: Fourth Revised Edition (Dover Books on Mathematics) 4th Edition.
8. H.A. Taha, Operations Research
9. J.G. Chakraborty and P.R. Ghosh. Linear Programming and Game Theory
10. P.K. Gupta and D.S. Hira, Operations Research
11. I. N. Herstein : Topics in Algebra.
12. K. Hoffman and R. Kunze : Linear Algebra.
13. B.C. Chatterjee : Linear Algebra.
14. D. W. Jordan and P. Smith (1998): Nonlinear Ordinary Equations- An Introduction to Dynamical Systems (Third Edition), Oxford Univ. Press.
15. L. Perko (1991): Differential Equations and Dynamical Systems, Springer Verlag.
16. F. Verhulst (1996): Nonlinear Differential Equations and Dynamical Systems, Springer.
17. V. I. Arnold : Dynamical Systems V-Bifurcation Theory and Catastrophe Theory.
18. Mark Kot (2001): Elements of Mathematical Ecology, Cambridge Univ. Press.

MATCB 2.1 is based on the choices of the students of Other Department(s)

MATC 2.2

Real Analysis–II

(Pure and Applied Streams)

Marks : 32 (SEE: 25; IA: 07)

The Lebesgue measure : Definition of the Lebesgue outer measure on the power set of \mathbb{R} , countable subadditivity, Carathéodory's definition of the Lebesgue measure and basic properties. Measurability of an interval (finite or infinite), Countable additivity, Characterizations of measurable sets by open sets, G_α sets, closed sets and F_σ sets. Measurability of Borel sets, Existence of non-measurable sets (8)

Measurable functions : Definition on a measurable set in \mathbb{R} and basic properties, Simple functions, Sequences of measurable functions, Measurable functions as the limits of sequences of simple functions, Lebesgue's theorem on restricted continuity of measurable functions, Egoroff's theorem, Convergence in measure (5)

The Lebesgue integral : Integrals of non-negative simple functions, The integral of non-negative measurable functions on arbitrary measurable sets in \mathbb{R} using integrals of non-negative simple functions, Monotone convergence theorem and Fatou's lemma, The integral of Measurable functions and basic properties, Absolute character of the integral, Dominated convergence theorem, Inclusion of the Riemann integral, Riesz-Fischer theorem on the completeness of the space of Lebesgue integrable functions. (8)

Lebesgue integrability of the derivative of a function of bounded variation on an interval. Descriptive characterization of the Lebesgue integral on intervals by absolutely continuous functions. (4)

References :

1. W. Rudin : Principles of Mathematical Analysis.
2. D. V. Widder : Laplace Transform.
3. H. L. Royden : Real Analysis.
4. B. K. Lahiri and K. C. Ray : Real Analysis.
5. A. G. Das : The Generalized Riemann Integral.
6. A. G. Das : Theory of Integration – The Riemann, Lebesgue and Henstock-Kurzweil Integrals.
7. W. Sierpinski : Cardinal Number and ordinal Number.
8. I. P. Natanson : Theory of Integrals of a Real Variable (Vol. I and Vol. II).

Complex Analysis – II
(Pure and Applied Streams)
Marks : 32 (SEE: 25; IA: 07)

Zeros of an analytic function. Singularities and their classification. Limit points of zeros and poles. Riemann's theorem. Weierstrass-Casorati theorem. Theory of residues. Argument principle. Rouché's theorem. Maximum modulus theorem. Schwarz lemma. Behaviour of a function at the point at infinity.

(15)

Contour integration. Conformal mapping, Bilinear transformation. Idea of analytic continuation.

(7)

Multivalued functions – branch point. Idea of winding number.

(3)

References :

1. A. I. Markushevich : Theory of Functions of a Complex Variable(Vol. I, II and III).
2. R. V. Churchill and J. W. Brown : Complex Variables and Applications.
3. E. C. Titchmarsh : The Theory of Functions.
4. E. T. Copson : An Introduction to the Theory of Functions of a Complex Variable.
5. J. B. Conway : Functions of One Complex Variable.
6. L. V. Ahlfors : Complex Analysis.
7. H. S. Kasana : Complex Variables – Theory and Applications.
8. Shanti Narayan and P. K. Mittal : Theory of Functions of a Complex Variable.
9. A. K. Mukhopadhyay : Functions of Complex Variables and Conformal Transformation.
10. J. M. Howie : Complex Analysis.

Functional Analysis – II
(Pure and Applied Streams)
Marks : 36 (SEE: 30; IA: 06)

Linear operators, Linear operators on normed linear spaces, continuity, bounded linear operators, norm of an operator, various expressions for the norm. Spaces of bounded linear operators. Inverse of an operator.

(8)

Linear functionals. Hahn-Banach theorem (without proof), simple applications. Normed conjugate space and separability of the space. Uniform boundedness principle, simple application.

(5)

Inner product spaces, Cauchy Schwarz's inequality, the induced norm, polarization identity, parallelogram law. (3)

Orthogonality, Pythagoras Theorem, orthonormality, Bessel's inequality and its generalisation. (4)

Hilbert spaces, orthogonal complement, projection theorem. The Riesz's representation theorem. Convergence of series corresponding to orthogonal sequence, Fourier coefficient, Parseval's identity. (10)

References :

1. E. Kreyszig : Introductory Functional Analysis with Applications.
 2. G. Bachman and L. Narici : Functional Analysis.
 3. W. Rudin : Functional Analysis.
 4. N. Dunford and L. Schwartz : Linear Operators (Part I).
 5. A. E. Taylor : Introduction to Functional Analysis.
 6. B. V. Limaye : Functional Analysis.
 7. K. Yoshida : Functional Analysis.
 8. B. K. Lahiri : Elements of Functional Analysis.
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MATC 2.1

Mechanics – II (Classical Mechanics)

(Pure and Applied Streams)

Marks : 32 (SEE: 25; IA: 07)

Lagrangian Formulation : Generalised coordinates. Holonomic and nonholonomic systems. Scleronomic and rheonomic systems. D'Alembert's principle. Lagrange's equations. Energy equation for conservative fields. Cyclic (ignorable) coordinates. Generalised potential. (7)

Moving Coordinate System : Coordinate systems with relative translational motions. Rotating coordinate systems. The Coriolis force. Motion on the earth. Effect of Coriolis force on a freely falling particle. Euler's theorem. Euler's equations of motion for a rigid body. Eulerian angles. (6)

Variational Principle : Calculus of variations and its applications in shortest distance, minimum surface of revolution, Brachistochrone problem, geodesic. Hamilton's principle. Lagrange's undetermined multipliers. Hamilton's equations of motion. (5)

Canonical Transformations : Canonical coordinates and canonical transformations. Poincaré theorem. Lagrange's and Poisson's brackets and their variance under canonical transformations, Hamilton's equations of motion in Poisson's bracket. Jacobi's identity. Hamilton-Jacobi equation. (4)

Small Oscillations : General case of coupled oscillations. Eigen vectors and Eigen frequencies. Orthogonality of Eigen vectors. Normal coordinates. Two-body problem. (3)

References :

1. E. T. Whittaker : A Treatise of Analytical Dynamics of Particles and Rigid Dynamics.
2. Greenwood : Dynamics.
3. F. Chorlton : Dynamics.
4. Routh : Dynamics.
5. H. Lamb : Dynamics.
6. R. G. Takwale and P. S. Puranik : Introduction to Classical Mechanics.
7. H. Goldstein : Classical Mechanics.

Abstract Algebra – II

(Pure and Applied Streams)

Marks : 32 (SEE: 25; IA: 07)

Preliminaries: Review of earlier related concepts-Rings, integral domains, fields and their simple properties. (3)

Detailed discussion on rings: Classification of rings, their definitions and characterization theorem with examples and counter examples. polynomial rings, division algorithm, irreducible polynomials, Eisenstein's criterion for irreducibility. (6)

Ideals in rings: Definitions, classifications with related theorems, examples and counter examples. (6)

Domains in rings: Classification, definitions and related theories with example and counter examples. (6)

Field extensions: Definition and simple properties. (4)

References :

1. I. N. Herstein – Topics in Algebra.
2. Malik, Mordeson and Sen – Fundamentals of Abstract Algebra.

3. M. R. Adhikari and Abhishek Adhikari – Groups, Rings and Modules with Applications.
4. S. Lang – Algebra.
5. J. B. Fraleigh – A First Course in Abstract Algebra.
6. N. Jacobson – Lecturers in Abstract Algebra.
7. B. C. Chatterjee – Abstract Algebra (Vol. 1).
8. J. A. Gallian – Contemporary Abstract Algebra.
9. T. W. Hungerford – Algebra.
10. Luthar and Passi – Algebra (Vol. 1).

Operations Research – II
(Pure and Applied Streams)
Marks : 36 (SEE: 30; IA: 6)

Sensitivity Analysis : Changes in price vector of objective function, changes in resource requirement vector, addition of decision variable, addition of a constraint. (6)

Parametric Programming : Variation in price vector, Variation in requirement vector. (4)

Replacement and Maintenance Models : Failure mechanism of items, General replacement policies for gradual failure of items with constant money value and change of money value at a constant rate over the time period, Selection of best item (6)

Dynamic Programming (DP) : Basic features of DP problems, Bellman's principle of optimality, Multistage decision process with Forward and Backward recursive relations, DP approach to stage-coach problems. (5)

Non-Linear Programming (NLP) : Lagrange Function and Multipliers, Lagrange Multipliers methods for nonlinear programs with equality and inequality constraints. (4)

Separable programming, Piecewise linear approximation solution approach, Linear fractional programming. (5)

References :

1. Linear Programming – G. Hadley.
2. Mathematical Programming Techniques – N. S. Kambo.
3. Nonlinear and Dynamic Programming – G. Hadley.
4. Operations Research – K. Swarup, P. K. Gupta and Man Mohan.
5. Operations Research – H. A. Taha.

6. Introduction to Operations Research – A. Frederick, F. S. Hillier and G. J. Lieberman.
7. Engineering Optimization : Theory and Practice – S. S. Rao.
8. Principles of Operations Research – Harvey M. Wagner.
9. Operations Research – P. K. Gupta and D. S. Hira.
10. Nonlinear and Mixed-Integer Optimization – Christodoulos A. Floudas.
11. Operations Research : Theory and Applications – J. K. Sharma.

*** MATC 2.2 AND MATC 2.3 are common to both the pure and applied streams**

APPLIED STREAM

MATA 2.4

Mechanics of Fluids

(Applied Stream)

Marks :60 (SEE: 50; IA: 10)

Kinematics : Real and ideal fluids. Streamlines and paths of particles. Steady and unsteady flows. Lagrange's and Euler's methods of description of fluid motion. Accelerations. Boundary surface. Irrotational and rotational motions. Equation of continuity. (10)

Equations of Motion : Lagrange's and Euler's equations of motion. Bernoulli's theorem. Cauchy's integrals. Impulsive action. (8)

Motion in Two Dimensions : Stream function. Sources, sinks and doublets. Images. Image of a source (sink) with regard to a plane and a sphere. Image of a doublet with regard to a sphere, Images in two dimensions. Milne-Thomson circle theorem. Blasius theorem. (8)

General Theory of Irrotational Motion : Flow and circulation. Cyclic and acyclic motions. Impulsive motion. Properties of irrotational motion. Kelvin's theorem of minimum kinetic energy. Motion of a sphere. Liquid streaming past a fixed sphere. Equations of motion of a sphere. (8)

Vortex Motion : Vortex motion and its simple properties. Motion due to circular and rectilinear vortices. Vortex pair and doublet. Karman vortex street. (8)

Viscous Liquid Motion : Stress components in real fluid. Rate of strain quadric. Stress analysis in fluid motion. Relation between stress and rate of strain. Navier-Stokes' equations. Plane Poiseuille and Couette flow between two parallel plates. Flow through tubes of uniform cross-sections in the form of circle, annulus, ellipse under constant pressure gradient. (8)

References :

1. W. W. Besant and A. S. Ramsey : A Treatise
2. on Hydrodynamics.
3. G. K. Batchelor : An Introduction to Fluid Mechanics.
4. F. Chorlton : Text Book of Fluid Dynamics.
5. L. D. Landau and E. M. Lipschitz : Fluid Mechanics.
6. R. K. Rathy : An Introduction to Fluid Dynamics.
7. H. Lamb : Hydrodynamics.

Stochastic Processes**(Applied Stream)****Marks : 40 (SEE: 30; IA: 10)**

Review of Probability: Random variables, conditional probability and independence, bivariate and multi-variate distributions, probability generating functions, characteristic functions, convergence concepts. (10)

Conditional Expectation: Conditioning on an event, conditioning on a discrete random variable, conditioning on an arbitrary random variable, conditioning on a sigma-field. (5)

The Random Walk: unrestricted random walk, types of stochastic processes, gambler's ruin problem, generalisation of the random walk model. (5)

Markov Chains: Definitions, Chapman-Kolmogorov equation, Equilibrium distributions, Classification of states, Long-time behaviour. Stationary distribution. Branching process. (5)

Stochastic process in continuous time: Poisson process and Brownian motion. (5)

References:

1. Modern Probability Theory: B. R. Bhat.
2. Elementary Probability Theory and Stochastic Processes: K. L. Chung.
3. An Outline of Statistical Theory (Vol 1 and 2): A. M. Goon, M. K. Gupta & B. Dasgupta.
4. An Introduction to Multivariate Statistical Analysis: T. W. Anderson.
5. Introduction to Stochastic Processes: Hoel, Port, Stone
6. Stochastic Processes: Sheldon M. Ross

PURE STREAM

MATP 2.4

Differential Geometry–II

(Pure Stream)

Marks : 50 (SEE: 40; IA: 10)

Curves in the plane and space, surfaces in three-dimension, Smooth surface, Tangents and derivatives, normal and orientability, Examples of surfaces. (10)

The first fundamental form, Length of curves on surfaces, Isometries of surfaces, Conformal mapping of surfaces, (10)

Curvature of surfaces, The second fundamental form, The Gauss and Weingarten map, Normal and geodesic curvatures, Parallel transport and covariant derivative. (10)

Gaussian, mean and principal curvatures, Gauss Theorema Egregium, Minimal surface, The Gauss Bonnet Theorem. Abstract differentiable manifolds and examples, Tangent Spaces (10)

References :

1. I. S. Sokolnikoff : Tensor Analysis, Theory and Applications to Geometry and Mechanics of Continua.
2. Andrew Pressley, Elementary Differential Geometry.
3. L. P. Eisenhart : An Introduction to Differential Geometry (with the use of Tensor Calculus).
4. T. Y. Thomas : Concepts from Tensor Analysis and Differential Geometry.
5. U. C. De : Differential Geometry of Curves and Surfaces in E^3 (Tensor Approach).
6. Riemannian Geometry, M. P. Do Carmo

Topology–I1

(Pure Stream)

Marks : 50 (SEE: 40; IA: 10)

Connectedness : Examples, various characterizations and basic properties. Connectedness on the real line. Components and quasi components. Path connectedness and path components. (10)

Compactness : Characterizations and basic properties of compactness, Lebesgue, lemma. Sequential compactness, BW Compactness and countable compactness. Local compactness and Baire Category Theorem. (10)

Identification spaces: Constructing a Mobius strip, identification topology, Orbit spaces. (8)

Some Matrix Lie Groups : Some elementary properties of topological groups, $Gl(n, R)$ as a topological group and its subgroups. (10)

Fundamental groups, calculation of fundamental group of S^1 . (2)

References :

1. M. A. Armstrong, Basic Topology, Springer (India), 2004,
 2. J.R. Munkres, Topology, 2nd Ed., PHI (India), 2002,
 3. J. M. Lee : Introduction to topological Manifolds,
 4. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, New York, 1963.
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III. The Third Semester

MATC 3.1

Linear Algebra

(Pure and Applied Streams)

Marks : 40 (SEE: 30; IA: 10)

Matrices over a field: Matric polynomial, characteristic polynomial, eigen values and eigen vectors, minimal polynomial. (6)

Linear Transformation (L.T.): Definition and the algebra of L.T., Rank and Nullity of L.T., Dual space, dual basis, Representation of L.T. by matrices, Change of basis. (8)

Normal forms of matrices: Diagonalization of matrices, Smith's normal form, Invariant factors and elementary divisors, Jordan canonical form, Rational (or Natural Normal) form, triangular forms, (10)

Bilinear and Quadratic forms: Bilinear forms, quadratic forms, reduction and classification of quadratic forms. (6)

References :

1. I. N. Herstein : Topics in Algebra.
2. K. Hoffman and R. Kunze : Linear Algebra.
3. B.C. Chatterjee : Linear Algebra.
4. J. H. Kwak and S. Hong : Linear Algebra.
5. E. D. Nering : Linear Algebra and Matrix Theory.

Special Functions

(Pure and Applied Streams)

Marks : 25 (SEE: 20; IA: 05)

Solutions of Hypergeometric, Bessel, Legendre, Hermite differential equations. (4)

Legendre polynomial : Generating relation, Recurrence relations, Rodrigue's formula, Schlafli's and Laplace's integral formulae, Orthogonal property, Reconstruction of the Legendre differential equations. (4)

Hermite and Laguerre polynomials : Generating relations, Recurrence relations, Rodrigue's formulae, Orthogonal properties, Reconstructions of the respective differential equations. (6)

Chebyshev polynomial : Definition, Series representation, Recurrence relations, Deduction of Chebyshev differential equation, Orthogonal property. (2)

Bessel's functions : Generating relation for integral index, Recurrence relations, Representations for the indices $\frac{1}{2}$ and $-\frac{1}{2}$, Bessel's integral Formulae, Bessel's function of second kind. (4)

References :

1. E. T. Copson : An Introduction to the Theory of Functions of a Complex Variable.
2. N. N. Lebedev : Special Functions and their Applications.
3. I. N. Sneddon : Special Functions of Mathematical Physics and Chemistry.
4. E. D. Rainville : Special Function.

Integral Equations & Integral Transforms

(Pure and Applied Streams)

Marks : 35 (SEE: 30; IA: 05)

Integral Equations. Definitions of integral equations and their classification. Volterra integral equations of second kind : Solution by successive approximations. Resolvent kernel and solutions of Volterra integral equations. Fredholm integral equations of second kind : Resolvent kernel, solution in terms of resolvent kernel, solution with separable kernels, Method of successive approximations, iterative scheme for Fredholm integral equations. (7)

Classical Fredholm theory : Fredholm theorems, Fredholm Alternative Principles. (3)

Hilbert-Schmidt theory : Symmetric kernels, Orthogonal system of functions, Fundamental properties of eigenvalues and eigenfunctions for symmetric kernels, Hilbert-Schmidt theorem. (5)

Integral Transforms. Laplace Transform : Definition and basic properties. Laplace integral. Lerch's theorem (statement only). Laplace transforms of elementary functions, of derivatives and Dirac-delta function. Differentiation and integration. Convolution. Inverse transform. Applications to solve ordinary differential equations. (5)

Fourier Transform : Definition and basic properties. Fourier transform of some elementary functions, of derivatives. Inverse Fourier transform. Convolution theorem and Parseval's relation. Applications of Fourier inversion and convolution theorems. Fourier sine and cosine transforms. (5)

Hankel Transform : Definition and inversion formula. Hankel transform of derivatives. Finite Hankel transform. (2)

Applications : Applications of integral transforms to solve two-dimensional Laplace and one-dimensional diffusion and wave equations. (3)

References :

1. S. G. Michelins : Linear Integral Equations.
2. I. G. Petrovsky : Lectures on the Theory of Integral Equations.
3. R. P. Kanwal : Linear Integral Equations
4. D. V. Wider : The Laplace Transforms.
5. H. S. Carslaw and J. C. Jaeger : Operational Methods in Applied Mathematics.
6. N. V. McLachlan : Operational Calculus.
7. R. V. Churchill : Operational Mathematics.
8. I. N. Sneddon : The Use of Integral Transforms.

***MATC 3.1 is common to both the pure and applied stream**

APPLIED STREAM

MATA 3.2

Fuzzy Set Theory

(Pure and Applied Streams)

Marks : 26 (SEE: 20; IA: 06)

Interval Arithmetic: Interval numbers, arithmetic operations on interval numbers, distance between intervals, two level interval numbers. (2)

Basic concepts of fuzzy sets: Types of fuzzy sets, α -cuts and its properties, representations of fuzzy sets, decomposition theorems, support, convexity, normality, cardinality, standard set-theoretic operations on fuzzy sets, Zadeh's extension principle. (8)

Fuzzy Relations: Crisp versus fuzzy relations, fuzzy matrices and fuzzy graphs, composition of fuzzy relations, relational join, binary fuzzy relations. (4)

Fuzzy Arithmetic: Fuzzy numbers, arithmetic operations on fuzzy numbers (multiplication and division on \mathbb{R}^+ only), fuzzy equations. (6)

References :

1. Fuzzy Sets and Fuzzy Logic *Theory and Applications* – G.J. Klir and B. Yuan.
2. Introduction to Fuzzy Arithmetic *Theory and Applications* – A. Kaufmann and M.M. Gupta.
3. Fuzzy Set Theory – R. Lowen.
4. Fuzzy Set Theory and Its Applications – H.-J. Zimmermann.
5. Fuzzy Set, Fuzzy Logic, Applications – G. Bojadziev and M. Bojadziev.

Computer Programming in ‘C’ (Theory)

(Pure and Applied Streams)

Marks : 37 (SEE: 30; IA: 07)

Introduction and a Brief History of ‘C’ Language. (1)

Fundamentals of ‘C’ Language : Basic structure of a ‘C’ program, Basic Data type, Constants and Variables, Identifier, Keywords, Constants, Basic data type, Variables, Declaration and Initialization, Statements and Symbolic constants. Compilation and Execution of a ‘C’ program. (3)

Operators and Expressions : Arithmetic, Relational, Logical operators. Increment, Decrement, Control, Assignment, Bitwise, and Special operators. Precedence rules of operators, Type Conversion (casting), Modes of arithmetic expressions, Conditional expressions. (4)

Input / Output Operations : Formatted I/O - Single character I/O (getchar(), putchar()), Data I/O (scanf(), printf()), String I/O (gets(), puts()). Programming problems. (2)

Decision Making Statements : Branching – *if* Statement, *ifelse* Statement, Nested *if.... else* Statement. *elseif* and *switch* Statements.

Loop Control : *for* Statement, *while* Statement, *do while* Statement. *break*, *continue* and *exit* Statements. Programming problems. (4)

Functions : Function declaration, Library functions, User defined function, Passing argument to a function, Recursion. Programming problems. (3)

Arrays : Array declaration and static memory allocation. One dimensional, two dimensional and multidimensional arrays. Passing arrays to functions. Sparse matrix. (2)

Pointers : Basic concepts of pointer, Functions and Pointers. Pointers and Arrays, Memory allocation, Passing arrays to functions, Pointer type casting. Programming problems. (3)

Structures and Unions : Declaring a Structure, Accessing a structure element, Storing methods of structure elements, Array of structures, Nested structure, Self –referential structure, Dynamic memory allocation, Passing arrays to function. Union and rules of Union. Programming problems. (4)

File Operations : File Input / Output operations – Opening and Closing a file, Reading and Writing a file. Character counting, Tab space counting, File-Copy program, Text and Binary files. (4)

References :

1. Programming in ANSI C : E. Balaguruswamy.
2. Let Us C : Y. Kanetkar.
3. Programming in C Language : B. S. Gottfred.
4. Mastering Algorithm in C : K. Loudon.
5. The C Programming Language : B.W. Kernighan and D. Ritchie.

Numerical Analysis (Practical)

(Applied Streams)

Marks : 37 (SEE: 30; IA: 07)

(*Laboratory Note Book: 5 marks + Viva-Voce: 7 marks
+ Compilation and Execution of Two Problems).

Numerical Computation

- (i) Interpolation and Approximation :
 - a) cubic spline interpolation,
 - b) Least square approximation.
- (ii) Numerical Integration : (a) Gaussian quadrature, (b) Romberg formula.
- (iii) Eigen value and Eigenvector Problems : Power method.
- (iv) Solutions of Non-linear Equations : Newton-Raphson method.
- (v) Numerical Solutions of Ordinary Differential Equations for Initial Value Problems : (a) Euler's method, (b) Runge-Kutta method, (c) Milne's predictor–corrector method.

Practical Examination Related Criteria :

- (i) Laboratory Clearance be taken by the students prior to commencement of Practical Examination.
- (ii) The Lab. Assignment Dissertations of the students be submitted prior to commencement of Practical Examination.

- (iii) Duration of Practical Examination will be 3 (Three) hours.
- (iv) One External Examiner be appointed for Practical Examination.

References :

1. Balagurusamy, E. – Programming in ANSI C
2. Y. Kanetkar – Let Us C
3. B. S. Gottfred – Programming in C Language
4. C. K. Loudon – Mastering Algorithm in.
5. B.W. Kernighan and D. Ritchie – The C Programming Language
6. N. Kalicharan – C by Example
7. F. Scheid – Theory and Problems of Numerical Analysis.
8. C. Xavier – C Language and Numerical Methods.
9. E. Balagurusamy – Computer Oriented Statistical and Numerical Methods.
10. D. C. Sanyal, and K. Das – A Text Book of Numerical Analysis.
11. A. K. Mukhopadhyay – Introduction to Numerical Methods with Computer Programming.
12. M. K. Jain, S. R. K. Iyengar and R. K. Jain, – Numerical Methods for Scientific and Engineering Computation.

MATA 3.3

Dynamical Systems

(Applied Stream)

Marks : 50 (SEE: 40; IA: 10)

Autonomous and non-autonomous systems : Orbit of a map, fixed point, equilibrium point, periodic point, circular map, configuration space and phase space. (8)

Nonlinear oscillators-conservative system. Hamiltonian system. Various types of oscillators in nonlinear system viz. simple pendulum, and rotating pendulum. (5)

Limit cycles : Poincaré-Bendixon theorem (statement only). Criterion for the existence of limit cycle for Liénard's equation. (4)

Stability : Definition in Liapunov sense. Routh-Hurwitz criterion for nonlinear systems. Liapunov's criterion for stability. Stability of periodic solutions. Floquet's theorem. (10)

Solutions of nonlinear differential equations by perturbation method : Secular term. Nonlinear damping. Solutions for the equations of motion of a simple pendulum, Duffing and Vander Pol oscillators. (5)

Bifurcation Theory : Origin of Bifurcation, Bifurcation Value, Normalisation, Resonance, Stability of a fixed point. Bifurcation of equilibrium solutions – the saddle node bifurcation, the pitch-fork bifurcation, Hopf-bifurcation. (5)

Randomness of orbits of a dynamical system : The Lorentz equations, Chaos, Strange attractors. (3)

References :

1. D. W. Jordan and P. Smith : Nonlinear Ordinary Differential Equations.
2. F. Verhulst : Nonlinear Differential Equations and Dynamic Systems.
3. R. L. Davaney : An Introduction to Chaotic Dynamical Systems.
4. P. G. Drazin : Non-linear Systems.
5. K. Arrowsmith : Introduction to Dynamical Systems.
6. C. Havyshi : Nonlinear Oscillations in Physical Systems.
7. A. H. Nayfeh and D. T. Mook : Nonlinear Oscillations.
8. V. I. Arnold : Dynamical Systems V-Bifurcation Theory and Catastrophy Theory.
9. V. I. Arnold : Dynamical Systems III – Mathematical Aspects of Classical and Celestial Mechanics.

Numerical Analysis (Theory)

(Applied Stream)

Marks : 50 (SEE: 40; IA: 10)

Interpolation : Hermite's interpolation. Interpolation by iteration – Aitken's and Neville's schemes. (5)

Approximation of Function : Least square approximation. Weighted least square approximation. Orthogonal polynomials, Gram – Schmidt orthogonalisation process, Chebysev polynomials, Mini-max polynomial approximation. (5)

Numerical Integration : Gaussian quadrature formula and its existence. Euler- MacLaurin formula. Gregory-Newton quadrature formula. Romberg integration. (6)

Systems of Linear Algebraic Equations : Direct methods, Factorization method. (4)

Eigen value and Eigenvector Problems : Direct methods, Iterative method – Power method . (4)

Nonlinear Equations : Fixed point iteration method, convergence and error estimation. Modified Newton-Raphson method, Muller's method, Inverse interpolation method, error estimations and convergence analysis. (6)

Ordinary Differential Equations : Initial value problems – Picard's successive approximation method, error estimation. Single-step methods – Euler's method and Runge-Kutta method, error estimations and convergence analysis. Multi-step method – Milne's predictor-corrector method, error estimation and convergence analysis. (6)

Partial Differential Equations: Finite difference methods for Elliptic and Parabolic differential equations. (4)

References :

1. Froberg, C. E. – Introduction to Numerical Analysis.
2. Hildebrand, F.B. – Introduction to Numerical Analysis.
3. Ralston, A. and Rabinowits, P. – A First Course in Numerical Analysis.
4. Atkinson, K. and Cheney, W. – Numerical Analysis.
5. David, K. and Cheney, W. – Numerical Analysis.
6. Jain, M. F., Iyenger, S. R. K. and Jain, R.K. – Numerical Methods for Scientific and Engineering Computation.
7. Scheid, F. – Numerical Analysis.
8. Powell, M. – Approximation Theory and Methods.
9. Press, W. H., Flannery, D. P. , Tenkolsky S. K., and Venner Link W. T.– Numerical Recipes.
10. Sanyal, D. C. and Das, K. - A Text Book of Numerical Analysis.
11. Rajaraman, V. – Computer Oriented Numerical Methods.
12. Balagurusamy, E. – Computer Oriented Numerical Methods.
13. Reddy, J. N. – An Introduction to Finite Element Methods.
14. Sastry, S. S. – Introductory Methods of Numerical Analysis.

MATA 3.4

Mathematical Biology

(Applied Stream)

Marks : 70 (SEE: 55; IA: 15)

Effect of Nutrients on autotrophy-herbivore interaction: Introduction, Models on nutrient recycling and its stability, Effect of nutrients on autotrophy herbivore stability, Models on herbivore nutrient recycling on autotrophic production. (7)

Dynamics of Phytoplankton-Zooplankton system: Introduction, Models on phytoplankton-zooplankton system and its stability, Bio-control in plankton models with nutrient recycling. (7)

Microbial population model: Microbial growth in a chemostat. Stability of steady states. Growth of microbial population. Product formation due to microbial action. Competition for a growth- rate limiting substrate in a chemostat. (7)

Mathematical models in ecology: Discrete and Continuous population models for single species. Logistic models and their stability analysis. Lag factor and stability of population steady states. (7)

Continuous models for two interacting populations: Lotka-Volterra model of predator -prey system, Kolmogorov model. Trophic function. Gauss's Model. Analysis of predator-prey model with in limit cycle behavior, parameter domains of stability. Nonlinear oscillations in predator-prey system. (12)

Continuous models for three or more interacting populations: Food chain models. Stability of food chains. Species harvesting in competitive environment, Economic aspects of harvesting in predator-prey systems. (10)

Interaction of Ratio-dependent models: Introduction, May's model, ratio-dependent models of two interacting species, two prey- one predator system with ratio-dependent predator influence- its stability and persistence. (5)

References :

1. K. E. Watt : Ecology and Resource Management-A Quantitative Approach.
2. R. M. May : Stability and Complexity in Model Ecosystem.
3. Y. M. Svirzhev and D. O. Logofet : Stability of Biological Communities.
4. A. Segel : Modelling Dynamic Phenomena in Molecular Biology.
5. J. D. Murray : Mathematical Biology. Springer and Verlag.
6. N. T. J. Bailey : The Mathematical Approach to Biology and Medicine.
7. L. Perko (1991): Differential Equations and Dynamical Systems, Springer Verlag.

8. F. Verhulst (1996): Nonlinear Differential Equations and Dynamical Systems, Springer Verlag.
9. H. I. Freedman - Deterministic Mathematical Models in Population Ecology.
10. Mark Kot (2001): Elements of Mathematical Ecology, Cambridge Univ. Press

Electromagnetic Theory

(Applied Stream)

Marks : 30 (SEE: 25; IA: 05)

Electrostatics : Law of force, Electrostatic potential, Gauss' Law. Conductors and dielectrics. Energy of the electrostatic field. Electric dipoles. Double layers. Capacitors, Polarization, Electric displacements and energy. (3)

Steady currents: Current vector, ohm's law, Differential equations of the field and flow. (2)

Magnetostatics: Lorentz force, Magnetic induction. Biot-Savart law. Laws of magnetostatics. Ampere's law. Magnetic potentials. Magnetic dipole. Magnetic media. Magnetization current, Equations of the magnetic field. Boundary conditions. (8)

Maxwell's equations: Equations of continuity for time – varying fields, Maxwell equations. Boundary conditions. Maxwell's stress. (3)

Electromagnetic waves: Wave equation. Plane waves in a uniform non- conducting medium, Polarization. Electromagnetic energy and Poynting's theorem. Energy flux in a plane wave. Plane waves in a conducting medium. Reflection and refraction at a dielectric boundary, Fresnel relations. Reflection from a conductor- normal incidence. (5)

Relativistic electrodynamics: The principle of relativity. Lorentz transformation, Transformation of electrodynamics variables. Theory of special relativity (statement of the principles only). Transform relations for systems in relative motion, Derivation of electromagnetic relations. (4)

References :

1. Jackson, J, D – Classical Electrodynamics.
2. Hallen, E. – Electromagnetic Theory.
3. Jaens, J, H – Mathematical Theory of Electricity and Magnetism.
4. Jones, D. S – The Theory of Electromagnetism.
5. Sommerfield, A. – Electrodynamics
6. Landan, L. D and Lifshitz, E. M – The classical Theory of Fields.

7. Coulson, C. A and Boyd, T. J. M. – Electricity.
8. Cullwick, E. G. – Electromagnetism and Relativity.
9. Smith, J, H. – Introduction to Special Relativity.
10. Barut, A. O. – Electrodynamics and classical Theory of Fields and Particles.

PURE STREAM

MATP 3.2

Fuzzy Set Theory

(Pure and Applied Streams)

Marks : 26 (SEE: 20; IA: 06)

Interval Arithmetic: Interval numbers, arithmetic operations on interval numbers, distance between intervals, two level interval numbers. (2)

Basic concepts of fuzzy sets: Types of fuzzy sets, α -cuts and its properties, representations of fuzzy sets, decomposition theorems, support, convexity, normality, cardinality, standard set-theoretic operations on fuzzy sets, Zadeh's extension principle. (8)

Fuzzy Relations: Crisp versus fuzzy relations, fuzzy matrices and fuzzy graphs, composition of fuzzy relations, relational join, binary fuzzy relations. (4)

Fuzzy Arithmetic: Fuzzy numbers, arithmetic operations on fuzzy numbers (multiplication and division on \mathbb{R}^+ only), fuzzy equations. (6)

References :

6. Fuzzy Sets and Fuzzy Logic *Theory and Applications* – G.J. Klir and B. Yuan.
7. Introduction to Fuzzy Arithmetic *Theory and Applications* – A. Kaufmann and M.M. Gupta.
8. Fuzzy Set Theory – R. Lowen.
9. Fuzzy Set Theory and Its Applications – H.-J. Zimmermann.
10. Fuzzy Set, Fuzzy Logic, Applications – G. Bojadziev and M. Bojadziev.

Computer Programming in 'C' (Theory)

(Pure and Applied Streams)

Marks : 37 (SEE: 30; IA: 07)

Introduction and a Brief History of 'C' Language. (1)

Fundamentals of 'C' Language : Basic structure of a 'C' program, Basic Data type, Constants and Variables, Identifier, Keywords, Constants, Basic data type, Variables, Declaration and Initialization, Statements and Symbolic constants. Compilation and Execution of a 'C' program. (3)

Operators and Expressions : Arithmetic, Relational, Logical operators. Increment, Decrement, Control, Assignment, Bitwise, and Special operators. Precedence rules of operators, Type Conversion (casting), Modes of arithmetic expressions, Conditional expressions. (4)

Input / Output Operations : Formatted I/O - Single character I/O (getchar(), putchar()), Data I/O (scanf(), printf()), String I/O (gets(), puts()). Programming problems. (2)

Decision Making Statements : Branching – *if* Statement, *ifelse* Statement, Nested *if.... else* Statement. *elseif* and *switch* Statements.

Loop Control : *for* Statement, *while* Statement, *do while* Statement. *break*, *continue* and *exit* Statements. Programming problems. (4)

Functions : Function declaration, Library functions, User defined function, Passing argument to a function, Recursion. Programming problems. (3)

Arrays : Array declaration and static memory allocation. One dimensional, two dimensional and multidimensional arrays. Passing arrays to functions. Sparse matrix. (2)

Pointers : Basic concepts of pointer, Functions and Pointers. Pointers and Arrays, Memory allocation, Passing arrays to functions, Pointer type casting. Programming problems. (3)

Structures and Unions : Declaring a Structure, Accessing a structure element, Storing methods of structure elements, Array of structures, Nested structure, Self –referential structure, Dynamic memory allocation, Passing arrays to function. Union and rules of Union. Programming problems. (4)

File Operations : File Input / Output operations – Opening and Closing a file, Reading and Writing a file. Character counting, Tab space counting, File-Copy program, Text and Binary files. (4)

References :

6. Programming in ANSI C : E. Balaguruswamy.

7. Let Us C : Y. Kanetkar.

8. Programming in C Language : B. S. Gottfred.
9. Mastering Algorithm in C : K. Loudon.
10. The C Programming Language : B.W. Kernighan and D. Ritchie.

Computer Programming in 'C' (Practical)

(Pure Streams)

Marks : 37 (SEE: 30; IA: 07)

(*Laboratory Note Book: 5 marks + Viva-Voce: 5 marks
+ Compilation and Execution of Two Problems**: 20 marks).

1. Basic Computation :

- (i) Summation of natural numbers up to a given number.
- (ii) Summation of odd / even numbers up to a given number.
- (iii) Evaluation of the factorial of a given number.
- (iv) Summation of all the digits of a number.
- (v) Determination of the mean, variance and standard deviation from a list of numbers. (6)

2. Number Testing [Hints are provided] :

- (i) Generation of all the terms of Fibonacci Series up to a certain number.
(Hints : General term in Fibonacci Series is as follows :

$$F[i] = i, \text{ if } i < 2$$

$$= F[i - 1] + F[i - 2], \text{ if } i \geq 2$$
 (The resultant series is : 0,1,1,2,3,5,8,13,21,34,55 etc.)
- (ii) Testing of whether a number is prime or not.
- (iii) Checking whether a number is Armstrong number or not (Hints : A number is Armstrong if sum of the cubes of its digits, matches with the number – e.g., $153 = 1^3 + 5^3 + 3^3$).
- (iv) Checking whether a number is Peterson number or not (Hints : A number is Peterson if sum of the factorials of its digits, matches with the number – e.g., $145 = 1! + 4! + 5!$).
- (v) Checking whether a number is Perfect number or not (Hints : a number is Perfect if sum of the factors (except itself), matches with the number- e.g. $28 = 1 + 2 + 4 + 7 + 14$). (8)

3. Series Computation :

- (i) The Exponential Series : $e^x (= 1 + x + x^2/2! + x^3/3! \dots \text{up to } n \text{ terms})$.
- (ii) The base of a Natural Log : $e (= 1 + 1/1! + 1/2! + 1/3! \dots \text{up to } n \text{ terms})$.
- (iii) The Sine Series : $\sin(x) (= x - x^3/3! + x^5/5! - x^7/7! + \dots \text{up to } n \text{ terms})$.
- (iv) The roots of a quadratic equation : $ax^2 + bx + c = 0$ for any input a, b, c. (8)

4. Matrix Operation :

- (i) Matrix Addition and Matrix Multiplication using 2D Array.
- (ii) Matrix Inversion using 2D Array.
- (iii) Sorting of a list of numbers.
- (iv) Finding of the Amplitude, Modulus, Addition and Subtraction of Complex numbers using Structure. (8)

****Applications of Branches, Loops, Arrays and Structures mainly be taken into account in Lab. Assignment.**

Practical Examination Related Criteria :

- (i) Laboratory clearance be taken by the students prior to commencement of Practical Examination.
- (ii) The Lab. Assignment Dissertations of the students be submitted prior to commencement of Practical Examination.
- (iii) Duration of practical examination will be 3 (Three) hours.
- (iv) One External Examiner be appointed for Practical Examination.

References :

- 1. Programming in ANSI C : E. Balaguruswamy.
- 2. Let Us C : Y. Kanetkar.
- 3. Programming in C Language : B. S. Gottfred.
- 4. Mastering Algorithm in C : K. Loudon.
- 5. The C Programming Language : B.W. Kernighan and D. Ritchie.
- 6. C by Example : N. Kalicharan.

MATP 3.3

Topological Groups

(Pure Stream)

Marks : 50 (SEE: 40; IA: 10)

Definition of topological group and examples. Right and left translations. Homogeneity property in a topological group. Fundamental neighbourhood system of the identity element. Separation axioms in topological groups. (12)

Subgroups. Criteria to be Hausdorff subgroups. Invariant subgroups and open and closed subgroups. Compactly generated subgroups. Coset space and Quotient space. Natural mapping. Isomorphism theorems. (12)

Uniform structure of a topological group. Locally compact topological group and its basic properties. Properties of topological groups involving connectedness. Invariant pseudo-metrics. Character groups. (16)

References :

1. T. Husain : Introduction to Topological Groups.
2. D. Montgomery and L. Zippin : Topological Transformation Groups.
3. E. Hewitt and K. A. Ross : Abstract Harmonic Analysis (Vol. I).
4. L. Pontryagin : Topological Groups.
5. F. J. Higgins : An Introduction to Topological Groups.

Measure Theory

(Pure Stream)

Marks : 50 (SEE: 40; IA: 10)

Measures: Class of Sets, Measures, The extension Theorems and Lebesgue-Stieljes measures, Caretheodory extension of measure, Completeness of measure (6)

Integrations: Measurable transformations, Induced measures, distribution functions, Integration, More on Convergency, Product of two measure spaces. Fubini's theorem. (14)

L^p -spaces: L^p -Spaces, Dual spaces, Banach and Hilbert spaces. (14)

Differentiations: The Lebesgue-Radon-Nikodymtheorem, Signed measures, Product Measures (6)

References :

1. Measure Theory: K. B. Athreya and S. Lahiri,
2. Introduction to Probability and Measure: Parthasarathi,
3. Real analysis, Mordern Techniques and their applications: G. B. Folland,
4. Measure Theory: P. R. Halmos

MATP 3.4

Calculus of \mathbb{R}^n

(Pure Stream)

Marks : 50 (SEE: 40; IA: 10)

Differentiation on \mathbb{R}^n : Directional derivatives and continuity, the total derivative and continuity, total derivative in terms of partial derivatives, the matrix transformation of $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$. The Jacobian matrix.

(5)

The chain rule and its matrix form. Mean value theorem for vector valued function. Mean value inequality.

(3)

A sufficient condition for differentiability. A sufficient condition for mixed partial derivatives.

(2)

Functions with non-zero Jacobian determinant, the inverse function theorem, the implicit function theorem as an application of Inverse function theorem. Extremum problems with side conditions – Lagrange's necessary conditions as an application of Inverse function theorem.

(15)

Integration on \mathbb{R}^n : Integral of $f : A \rightarrow \mathbb{R}$ when $A \subset \mathbb{R}^n$ is a closed rectangle. Conditions of integrability.

Integrals of $f : C \rightarrow \mathbb{R}$, $C \subset \mathbb{R}^n$ is not a rectangle, concept of Jordan measurability of a set in \mathbb{R}^n .

Fubini's theorem for integral of $f : A \times B \rightarrow \mathbb{R}$, $A \subset \mathbb{R}^n$, $B \subset \mathbb{R}^n$, are closed rectangles.

Fubini's theorem for $f : C \rightarrow \mathbb{R}$, $C \subset A \times B$.

Formula for change of variables in an integral in \mathbb{R}^n .

(15)

References :

1. T. M. Apostol : Mathematical Analysis.
2. M. Spivak : Calculus on Manifolds.

Operator Theory

(Pure Stream)

Marks : 50 (SEE: 40; IA: 10)

Conjugate (or dual) spaces, determination of conjugate spaces of \mathbb{R}^n , ℓ_p for $1 \leq p < \infty$. Representation theorem for bounded linear functionals on $C[a, b]$ (Statement only), Conjugate spaces of $C[a, b]$ and some other spaces (results only).

(10)

Weak convergence and weak* convergence, characterization of weak convergence, sufficient condition for the equivalence of weak* convergence and weak convergence in the dual space. (6)

Canonical imbedding, reflexive spaces, connection between reflexivity and separability, embedding of $n. \ell.$ spaces into a Banach space, some consequences of reflexivity. (6)

Bounded linear operator, uniqueness theorem, adjoint of an operator and some properties. (3)

Self adjoint, compact, normal, unitary, projection, positive operators, square roots of positive operators : Characterizations and some of their basic properties. Conditions under which the sum of projections is also a projection, expression of the norm of self adjoint operator, invariant subspaces. Closed linear transformation, closed graph theorem and open mapping theorem. (15)

References :

1. C. Bachman and L. Narici : Functional Analysis.
 2. E. Kreyszing : Introductory Functional Analysis with Applications.
 3. W. Rudin : Functional Analysis.
 4. B. V. Limaye : Functional Analysis.
 5. B. K. Lahiri : Elements of Functional Analysis.
 6. P. K. Jain : Functional Analysis.
 7. G. F. Simons : Introduction to Topology and Analysis.
 8. A. E. Taylor : Introduction to Functional Analysis
 9. S. K. Berbarian : Introduction to Hilbert Spaces.
 10. A. N. Kolmogorov and S. V. Fomin : Elements of the Theory of Functions and Functional Analysis.
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IV. The Fourth Semester

MATC 4.1

Discrete Mathematics

(Applied and Pure Streams)

Marks : 65 (SEE: 50; IA: 10)

Graph Theory : Definition of (undirected) graphs, circuits, cycles, sub-graphs, induced sub-graphs. Degree of vertex. Connectivity. Planar graphs and their properties. Trees, Euler's formula for connected graphs. Complete and complete bipartite graphs. spanning trees. Fundamental cut set and cycles. Matrix representation of graphs, Kuratowski's theorem (statement only) and its use. Chromatic index ,chromatic numbers and stability numbers. (15)

Lattices : Lattices as partial ordered sets. Their properties. Lattices as algebraic system. Sublattices. Direct products and Homomorphism. Some special Lattices e.g. complete complemented and distributed lattices. (5)

Boolean Algebra : Basic Definitions, Duality, Basic theorems, Boolean algebra as lattices, Sum and Product of Boolean algebra Minimal Boolean Expressions, Prime implicants, Logic gates and circuits. Truth tables, Boolean functions. Applications of Boolean Algebra to Switching theory (using AND,OR & NOT gates). Karnaugh Map method. (12)

Combinatorics : Introduction, Basic counting principles, permutation and combination, pigeonhole principles, Recurrence relations and generating functions. (8)

Grammar and Language : Introduction, Alphabets, Words, Free semi group, Languages, Regular expression and regular languages. Finite Automata (FA). Grammars. Finite State Machine. Non-deterministic and deterministic FA.

Push Down Automation (PDA). Equivalence of PDAs and Context Free Languages (CFLs), Computable Functions. (10)

References :

1. J. P Tremblay and R. Manohar : Discrete Mathematical Structures with Applications to Computers.
2. J. L. Gersting : Mathematical Structures for Computer Sciences.
3. S. Lepschutz : Finite Mathematics.
4. S. Wiitala : Discrete Mathematics – A Unified Approach.
5. J. E. Hopcroft and J. D. Ullman : Introduction to Automata Theory, Languages and Computation.

6. C. L. Liu : Elements of Discrete Mathematics.
7. F. Harary : Graph Theory.
8. C. Berge : The Theory of Graphs and its Applications.
9. N. Deo : Graph Theory with Applications to Engineering and Computer Science.
10. K . D. Joshi : Foundation of Discrete Mathematics.
11. S. Sahani : Concept of Discrete Mathematics.
12. L. S. Levy : Discrete Structure in computer Science.
13. J. H. Varlist and R. M. Wilson : A course in Combinatorics.
14. J. E. Whitesitt : Boolean Algebra and its Applications.
15. G. E. Revesz : Introduction to Formal Languages.
16. G. Birkhoff and T. C. Bartee : Modern Applied Algebra.
17. K. L. P. Mishra and N. Chandrasekaran : Automata, Languages, and Computation.

Probability and Statistical Methods

(Applied and Pure Streams)

Marks : 40 (SEE: 30; IA: 10)

Fields and σ -fields of events. Probability as a measure. Random variables. Probability distribution. Expectation. Moments. Moment inequalities, Characteristic function. Inversion theorem. Convergence of sequence of random variables-weak convergence, strong convergence and convergence in distribution, continuity theorem for characteristic functions. Weak and strong law of large numbers. Lindeberg-Levy Central Limit Theorem. (8)

Definition and classification of stochastic processes. Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution. (3)

Sufficient statistics, Completeness. Methods of estimation-maximum likelihood and moment methods of estimation, consistent estimators. Confidence intervals. Tests of hypotheses: most powerful and uniformly most powerful tests, likelihood ratio tests. Large sample tests. Simple nonparametric tests for one- and two-sample problems, test for independence. (6)

Gauss-Markov models, Estimability of parameters, Best linear unbiased estimators, Tests for linear hypotheses and confidence intervals. Analysis of variance and covariance. Fixed and random effects models. (5)

Multivariate normal distribution, Hotelling's T-square and Wishart distribution (without derivation) and their properties. Distribution of quadratic forms. Dimension reduction techniques: Principal component analysis, Discriminant analysis, Canonical correlation. (5)

Life-testing models, reliability and hazard function, reliability of series and parallel systems. (3)

References:

1. Modern Probability Theory: B. R. Bhat.
 2. Elementary Probability Theory and Stochastic Processes: K. L. Chung.
 3. An Outline of Statistical Theory (Vol 1 and 2): A. M. Goon, M. K. Gupta & B. Dasgupta.
 4. An Introduction to Multivariate Statistical Analysis: T. W. Anderson.
 5. Linear Statistical Inference and its Applications: C. R. Rao.
 6. Mathematical Statistics: S. S. Wilks.
 7. Life-testing and Reliability Estimation: S. K. Sinha & B. K. Kale.
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Detailed Syllabi for the Optional Subjects with Covering of

MATO 4.2 and MATO 4.3 :

Optional Subjects for both Applied and Pure Streams

Optional Subject

Advanced Operations Research–I

(Applied and Pure Streams)

Marks : 100 (SEE: 80; IA -20)

Network Analysis– Network definitions, Minimal Spanning Tree Algorithm, Shortest Route Algorithms, Max-flow Min-cut theorem, Generalized Max-flow Min-cut theorem, linear programming interpretation of Max-flow Min-cut theorem, minimum cost flows. A brief introduction to PERT and CPM, Components of PERT/CPM Network and precedence relationships, Critical path analysis, PERT analysis in controlling project. (20)

Queueing Theory : Basic features of Queueing Systems, Operating characteristics of a Queueing System, Arrival and Departure (birth and death) distributions, Inter-arrival and service times distributions, Transient steady-state conditions in queueing process.

Poisson queueing models : $(M / M / 1) : (\infty / \text{FIFO} / \infty)$; $(M / M / 1) : (N / \text{FIFO} / \infty)$; $(M / M / C) : (\infty / \text{FIFO} / \infty)$; $(M / M / C) : (N / \text{FIFO} / \infty)$, $C \leq N$;

$(M / M / R) : (K / \text{GD} / K)$, $R < K$ – machine servicing model; (12)

Simulation : A brief introduction to simulation, Advantages of simulations over traditional search methods, Limitations of simulation techniques, Computational aspects of simulating a system, random number generation in stochastic simulation, Monte-Carlo simulation and modelling aspects of a system, Simulation approaches to inventory and queueing systems. (6)

Linear Multi-Objective Programming (LMOP) : Conversion of LMOP to linear programming, *Minsum* and *Priority based* Goal Programming (GP) approaches to LMOP problems, Fuzzy Set - Theoretic approaches to GP Problems. (6)

Hierarchical Decision Analysis : Introduction to Bilevel Programming (BLP) and Multilevel Programming (MLP), Fuzzy Programming approaches to BLP problems. (6)

Genetic Algorithms (GAs) : Introduction to GAs, Robustness of GAs over traditional search methods. Binary encodings of candidate solutions, Schema Theorem and Building Block Hypothesis, Genetic

operators – crossover and mutation, parameters for GAs, Reproduction mechanism for producing Offspring, Darwinian Principle in evaluating objective function, Simple GA schemes, GA approaches to optimization problems. (8)

Reference :

1. Operations Research – K. Swarup, P. K. Gupta and Man Mohan.
2. Operations Research – H. A. Taha.
3. Operations Research – S. D. Sharma.
4. Introduction to Operations Research – A. Frederick, F. S. Hillier and G. J. Lieberman.
5. Optimization Theory and Applications – S. S. Rao.
6. Engineering Optimization : Theory and Practice – S. S. Rao
7. Optimization Methods in Operation Research – K. V. Mital.
8. Inventory Control – J. Jonson and D. Montogomer.
9. Analysis of Inventory Systems – G. Haddly and T. M. Within.
10. Queuing Theory – J. A. Panico.
11. Introduction to Theory of Queues – L. Takacs.
12. Linear Programming in Single and Multiple Objective System – J. P. Ignizio.
13. Decisions with Multiple Objectives – R. L. Keeney and H. Raiffs.
14. Linear Goal Programming – M. J. Schniederjans.
15. Linear Multiobjective Programming – M. Zeleny.
16. Multi-objective Programming and Goal Programming : *Theory and Applications* – T. Tanino, T. Tanaka and M. Inuiguchi.
17. Multi-objective Programming and Goal Programming : *Theory and Applications* – M. Tamiz.
18. Goal Programming and Extensions – J. P. Ignizio.
19. Handbook of Critical Issues in Goal Programming – C. Romero.
20. Fuzzy Multiple Objective Decision Making – Y. J. Lai and C. L. Hwang.
21. Fuzzy Set Theory and its Applications – H. J. Zimmermann.
22. Genetic Algorithms in Search, Optimization and Machine Learning – D. E. Goldberg.
23. An Introduction to Genetic Algorithms – M. Mitchell.
24. Genetic Algorithms – K. F. Man, K. S. Tang and S. Kwong.
25. Genetic Algorithms + Data Structures = Evolution Programs – Z. Michalewicz.
26. Adaptation in Natural and Artificial Systems - J. H. Holland.

Optional Subject
Advanced Operations Research–II
(Applied and Pure Streams)
Marks : 100 (SEE: 80; IA -20)

Theory of Inventory Control : A brief introduction to Inventory Control, Single-item deterministic models without shortages and with shortages, models with price breaks. Dynamic Demand Inventory Models. (8)

Single-item stochastic models without Set-up cost and with Set-up cost (5)

Multi-item inventory models with the limitations on warehouse capacity, Average inventory capacity, Capital investment. (4)

Information Theory : Information concept, expected information, bivariate information theory, economic relations involving conditional probabilities, Entropy and properties of entropy function. (12)

Coding theory : Communication system, encoding and decoding, Shannon-Fano encoding procedure, Haffman encoding, noiseless coding theory, noisy coding, error detection and correction, minimum distance decoding, family of codes, Hamming code, Golay code, BCH codes, Reed-Muller code, perfect code, codes and design, Linear codes and their dual, weight distribution. (12)

Markovian Decision Process : Ergodic matrices, regular matrices, imbedded Markov Chain method for Steady State solution. (8)

Reliability : Elements of Reliability theory, failure rate, extreme value distribution, analysis of stochastically failing equipments including the reliability function, reliability and growth model. (8)

Geometric Programming (GP):

Posynomial, Signomial, Degree of difficulty, Unconstrained minimization problems, Solution using Differential Calculus, Solution seeking Arithmetic-Geometric inequality, Primal dual relationship & sufficiency conditions in the unconstrained case, Constrained minimization, Solution of a constrained Geometric Programming problem, Geometric programming with mixed inequality constraints, Complementary Geometric programming. (12)

References :

1. An Introduction to Information Theory – F. M. Reza.
2. Operations Research : *An Introduction* – P. K. Gupta and D.S. Hira.
3. Graph Theory with Applications to Engineering and Computer Science – N. Deo.

4. Operations Research –K. Swarup, P. K. Gupta and Man Mohan.
5. Coding and Information Theory – Steven Roman.
6. Coding Theory, A First Course – San Ling r choaping Xing.
7. Introduction to Coding Theory – J. H. Van Lint
8. The Theory of Error Correcting Codes – Mac William and Sloane.
9. Information and Coding Theory – Grenth A. Jones and J. Marry Jones.
10. Information Theory, Coding and Cryptography – Ranjan Bose.

Optional Subject

Fuzzy Sets and Systems

(Applied and Pure Streams)

Marks : 100 (SEE: 80; IA: 20)

- Fuzzy Sets:** From crisp sets to fuzzy sets: a shift of paradigm, preliminaries. (2)
- Operations on Fuzzy Sets:** Fuzzy complements, axioms of fuzzy complements, equilibrium, dual point, characterization theorem of fuzzy complements, increasing and decreasing generators. t-norms, t-conorms, their axioms and corresponding characterization theorems, dual triple. (10)
- Fuzzy Relations:** Fuzzy equivalence relations, fuzzy Compatibility relations, fuzzy ordering relations, Projections and cylindric extensions. (6)
- Fuzzy Arithmetic:** Linguistic variables, arithmetic operations on fuzzy numbers (On \mathbb{R} , in general). (4)
- Defuzzification of Fuzzy Numbers:** Definition, Different types of defuzzification techniques. (5)
- Fuzzy Logic:** A brief review of Classical logic, fuzzy propositions, fuzzy quantifiers, fuzzy inference rules, inferences from fuzzy propositions. (12)
- Possibility Theory:** Fuzzy measures, evidence theory, belief measures and plausibility measures, possibility theory, necessity measures, possibility measures, possibility distributions, fuzzy sets and possibility theory, possibility theory versus probability theory. (8)
- Fuzzy Decision Making:** Introduction to decision- making in Fuzzy environment. Individual decision making, multiperson decision making, multicriteria decision making, fuzzy ranking methods, fuzzy linear programming, multiobjective fuzzy programming. (10)
- Fuzzy Control :** Expert Systems, Expert-Knowledge representation techniques, Input and Output variables, Fuzzy controller, Inference engine (rule-firing), Fuzzification. Mamdani fuzzy control System, Takagi-Sugeno fuzzy control System. (8)

References :

1. The Importance of Being Fuzzy – A. Sangalli.
 2. Fuzzy Sets and Fuzzy Logic *Theory and Applications* – G. J. Klir and B. Yuan.
 3. Introduction to Fuzzy Arithmetic *Theory and Applications* – A. Kaufmann and M. M. Gupta.
 4. Fuzzy Sets and Systems – D. Dubois and H. Prade.
 5. Fuzzy Set Theory – R. Lowen.
 6. A First Course in Fuzzy Logic – H. T. Nguyen and E. A. Walker.
 7. Fuzzy Logic – J. E. Baldwin.
 8. Fuzzy Set Theory and Its Applications – H. J. Zimmermann.
 9. Fuzzy Multiple Objective Decision Making – Y. J. Lai and C. L. Hwang.
 10. Fuzzy Set, Fuzzy Logic, Applications – G. Bojadziev, M. Bojadziev.
 11. Fuzzy Control – S. S. Farinwata, D. Filev, and R. Langari.
 12. Fuzzy Logic for Planning and Decision Making – F. A. Lootsma.
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Optional Subjects for Only Applied Stream

Optional Subject

Advanced Solid Mechanics

(Applied Stream)

Marks : 100 (SEE: 80; IA: 20)

Elastostatics : Orthogonal curvilinear coordinates. Strain and rotation components, dilatation. Equations of motion in terms of dilatation and rotation. Stress equations of motion. Radial displacement. Spherical shell under internal and external pressures, gravitating sphere. Displacement symmetrical about an axis. Cylindrical tube under pressure, rotating cylinder. (15)

Problems of semi-infinite solids with displacements or stresses prescribed on the plane boundary. (4)

Variational methods. Theorems of minimum potential energy. Betti-Rayleigh reciprocal theorem. Use of minimum principle in the case of deflection of elastic string of central line of a beam. (6)

Equilibrium of thin plates. Boundary conditions. Approximate theory of thin plates. Application to thin circular plates. (6)

Elastodynamics: Waves in isotropic elastic solid medium. Surface waves, e.g. Rayleigh and Love waves. Kinematical and dynamical conditions in relation to the motion of a surface of discontinuity. Poisson's and Kirchoff's solutions of the characteristic wave equation. (8)

Radial and rotatory vibration of a solid and hollow sphere. Radial and torsional vibration of a circular cylinder. (6)

Transverse vibration of plates, Basic differential equations. Vibration of a rectangular plate with simply supported edges. Free vibration of a circular plate. (5)

Plasticity: Basic concepts and yield criteria. Prandtl-Reyss theory, Stress-strain relations of Von-Mises. Hencky's theory of small deformation. (5)

Torsion of cylindrical bars of circular and oval sections. Bending of a prismatic bar of narrow rectangular cross-section by terminal couple. Spherical and cylindrical shell under internal pressure. Plastic deformation of flat rings. (8)

Slip lines and plastic flow. Plastic mass pressed between two parallel planes. (4)

References :

1. Sokolnikoff I. S. : Mathematical Theory of Elasticity.

2. Love A.E. H. : A Treatise on the Mathematical Theory of Elasticity.
3. Fung Y.C. : Foundations of Solid Mechanics.
4. Timoshenko S. and Goodier N : Theory of Elasticity.
5. Ghosh. P.K. : Waves and Vibrations.
6. Prager, N and Hodge , P.G. : Theory of Perfectly Plastic Solids.
7. Southwell, R. V. : Theory of Elasticity.

Optional Subject
Advanced Fluid Mechanics
(Applied Stream)
Marks : 100 (SEE: 80; IA: 20)

Incompressible fluid: Elementary theory of aerofoils: Kutta - Joukowski's theorem. Joukowski's hypothesis. Joukowski's, Karmann-Trefftz and Mises family of profiles.

Theory of discontinuous potential motion. Kirchhof's method of solving problems of two-dimensional motion with free streamlines. Levi - Cevita's method. Concept of a vortex sheet. Karmann's vortex sheet and its stability. Karmann's formula for resistance due to a vortex wake. (15)

Prandtl boundary layer. Boundary layer equations. Blasius solution. Boundary layer parameters. (5)

Compressible fluid: Polytropic gas and its entropy. Adiabatic and isentropic flow. Propagation of small disturbance. Bernoulli's integral. Isentropic flow of a perfect gas. Subsonic and supersonic flow. Mach numbers and critical speeds. Mach lines. Normal and oblique shock waves. Steady isentropic irrotational flow. Prandtl - Meyer flow. Hodograph equations, characteristic of steady flow in the real and hodograph plane. (15)

Viscous flow: Navier-Stokes equations in orthogonal curvilinear coordinates. Dissipation of energy. Hydrodynamical theory of lubrication. Principle of similitude. Two – dimensional motion of viscous liquid (equation satisfied by the stream function). Hamel's equation and its solution. Diffusion of vorticity from a line vortex. Stokes and Lamb's solutions. Prandtl equation of boundary layer. Steady plane and circular jets. (15)

Turbulent flow: Mean values. Reynolds theory. Mixing length theories. Momentum transfer theory. Taylor's vorticity transfer theory. Karmann's similarity hypothesis. Applications to the solutions of (i) mixing zone between two parallel flows, (ii) motion in a plane jet. Prandtl 1/7 power law and its application to turbulent boundary layer over a flat- plate. (15)

References :

1. Goldstein, A : Modern Development in Fluid Mechanics (Vol. I & II).
2. Lamb, H. : Hydrodynamics.
3. Milne-Thomson, L. M . : Theoretical Hydrodynamics.
4. Pai, S. I. : Viscous Flow Theory (Vol. I & II).
5. Landau L. D. and Lifshitz E. M. : Fluid Mechanics.
6. Schlichting H. : Boundary Layer Theory.
7. Young , A. D. : Boundary Layers.
8. Batchelor, G. K. : An Introduction to Fluid Mechanics.
9. Pai, S. I. : Theory of Jets, Turbulent Flow.

Optional Subject

Computational Fluid Mechanics

(Applied Stream)

Marks : 100 (SEE: 80; IA: 20)

A brief Introduction to Computational Fluid Mechanics.

Stationary convection : Diffusion equation (finite volume discretization schemes of positive type, upwind discretization). (5)

Nonstationary convection : Diffusion equation: Stability. Discrete maximum principle. (5)

Incompressible Navier-Stokes (NS) equations : Boundary conditions. Spatial and temporal discretization on collocated and on staggered grids. (5)

Iterative method : Stationary methods. Krylov subspace methods. Multigrid methods. Fast position solvers. Iterative methods for incompressible NS equations. (15)

Shallow water equations : One - and two-dimensional cases. (5)

Scalar conservation laws : Godunov's order Barrier Theorem. Linear Schemes. (5)

Euler equation in one space dimension : Analytic aspects. Approximate Riemann solver. Osher scheme. Flux splitting schemes. Stability. James-Schmidt-Turkel scheme. Higher order scheme. (13)

Discretization in general domains : Boundary fitted grids. Equations of motion in general coordinates. Numerical solution of Euler equation in general coordinates. Numerical solution of NS equations in general domains. (10)

References:

1. Wesseling, P. : Principle of Computational Fluid Dynamics.
 2. Anderson, J. D. : Principle of Computational Fluid Dynamics; The Basics with Applications.
 3. Wendt, J. F., Anderson J. D., Degrez G. and Dick E. : Principle of Computational Fluid Dynamics.
 4. Ferziger, J. H. and Peric, M. : Computational Methods for Fluid Dynamics.
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Optional Subject
Magneto-Fluid Mechanics
(Applied Stream)
Marks : 100 (SEE: 80; IA: 20)

Fundamental equations: Maxwell's electromagnetic field equations. Basic Magneto-Fluid Dynamics (MFD) equations. Energy conservation equation. Equations for infinitely conducting medium. Lundquist equations. Properties of MFD equations, Magnetic Reynolds number. Boundary conditions. Alfven's wave. Magnetic body force. Ferraro's law of isorotation. (16)

Incompressible magneto-hydrodynamic flow : Parallel steady flow. One-dimensional steady viscous flow. Isentropic and homentropic flows. Hartmann and Couette flows. (12)

Characteristics of MFD waves : Characteristic equation. Characteristic determinant. Magneto hydrodynamic waves. Fast, slow, transverse and entropy waves. (15)

MFD shock waves, and Jump relation: The generalized Hugoniot condition. The compressive nature of magneto hydrodynamic shocks. Mach number, Subsonic and supersonic flows. Sub and super Alfvenic waves. (12)

MFD Stability: Normal mode analysis of stability for infinitely conducting, inviscid and incompressible medium. Rayleigh-Taylor and Kelvin -Helmholtz instabilities in presence of horizontal magnetic field. Capillary instability of a jet in presence of axial magnetic field. Stability of pinch. Principle of exchange instability – marginal stability analysis of a layer of fluid heated from below in presence of uniform magnetic field and gravity perpendicular to the boundary. (15)

References

1. Jeffrey, A.: Magneto Hydrodynamics.
 2. Cowling, T. G. : Magneto Hydrodynamics.
 3. Ferraro, V. C. A. and Plumpton. C. : An Introduction to Magnetofluid Mechanics.
 4. Pai, S. I. : Magnetogas Dynamics and Plasma Dynamics.
 5. Cramer, K. R. and Pai S. I. : Magnetofluid Dynamics for Engineers and Physicists.
 6. Shercliff, J. A. : Magnetohydrodynamics.
 7. Bansal, J. L. : Magnetofluid Dynamics of Viscous Fluids.
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Optional Subject

Plasma Physics

(Applied Stream)

Marks : 100 (SEE: 80; IA: 20)

Field of a moving point charge: Radiation from an accelerated charge. Radiation power. Damping force of radiation. Lagrangian and Hamiltonian for the motion of a charge particle in electromagnetic field. (10)

Non-relativistic motion: Non-relativistic motion of charged particles in electric and magnetic fields. Gradient and curvature drifts. (6)

Basic Plasma properties : Waves in unmagnetized and cold magnetized Plasmas. Radiation from plasma-the Bremsstrahlung and Synchrotron radiation. Stream instabilities in cold plasma. (15)

Collision processes in plasmas : Two-body elastic collisions. Two-particle Coulomb interaction. Thomson and Rayleigh scattering. Compton radiation. (8)

Small amplitude waves in plasmas: Linearized equations. Anisotropy of magnetized plasmas. Appleton-Hartree equation. Dielectric and conductivity tensors. Electromagnetic field in dissipative plasmas. (10)

Kinetic approach-Linearized Vlasov equations: Small amplitude Oscillations- Landau damping. (7)

Derivation of MHD equations : General properties, e.g. generalization of Bernoulli's and Kelvin's theorems, diamagnetic drifts and currents. Double-adiabatic theory for collisionless plasma- the Chew-Goldberger-low (CGL) equations. (7)

Space and astrophysical plasmas : Structuring of plasmas in solar system and magnetospheres. Magnetic reconnections. Double layers and particle acceleration. Solar wind-magnetosphere-Ionosphere intersection. Solar wind intersection with smaller bodies. (4)

Dusty plasmas : Dusty plasmas and the role of dust in stellar environment, galactic and planetary systems. (3)

References :

1. Jackson, J. D. : Classical Electrodynamics.
2. Jones, D. S. : Theory of Electromagnetism.
3. Landau, L. D. and Lifshitz E. M. : Classical Theory of Fields.
4. Panofsky, W. K. H. and Philips M. : Classical Theory of Fields.
5. Kompanoyets, A.S. : Theoretical Physics.
6. Alfven, H. and Falthamman, C. A. : Cosmical Electrodynamics.
7. Chandrasekher, S. : Plasma Physics.
8. Thomson, W.B. : An Introduction to Plasma Physics.
9. Clemmow, P.C. and Dougherty J. P. : Electrodynamics of Particles and Plasma.
10. Chakraborty, B. : Principles of Plasma Mechanics.

Optional Subject

Mathematics of Finance and Insurance

(Applied Stream)

Marks : 100 (SEE: 80; IA: 20)

Mathematics of Finance (SEE: 50; IA: 12)

Financial derivatives : An introduction. Types of financial derivatives – forwards and futures. Option and its kinds; and SWAPS. The Arbitrage Theorem and Introduction to Portfolio selection and capital Market Theory : Static and Continuous – Time model. (10)

Pricing by Arbitrage : A single –period option pricing model; Multi – period pricing model – Cox – Ross – Rubinstein model; Bounds on option prices. The Ito’s lemma and the Ito’s integral. (6)

Dynamics of derivative prices : Stochastic differential equations (SDEs) –Major models of SDEs, Linear constant coefficient SDEs, Geometric SDEs, Square root process, Mean reverting process and Ornstein- Uhlenbeck process. (6)

Martingale measures and risk-neutral probabilities : Pricing of binomial options with equivalent martingale measures. (6)

The Black-Scholes option pricing : Model with no arbitrage approach, limiting case of binomial option pricing and risk –neutral probabilities. (6)

The American Option pricing : Extended trading strategies. Analysis of American put options; early exercise premium and relation of free boundary problem. (6)

Mathematics of Insurance (SEE: 30; IA: 08):

Concepts from insurance : Introduction. The claim number process. The claim size process. Solvability of the portfolio. Reinsurance and ruin problem. (6)

Premium and ordering of risks : Premium calculation principles and ordering distributions. (5)

Distribution of aggregate claim amount : Individual and collective model. Compound distribution. Claim number of distribution. Recursive computation methods. Lundberg bounds and approximation by compound distributions. (8)

Risk processes: Time-dependent risk models. Poisson arrival processes. Ruin probabilities and bounds asymptotic and approximation. (5)

Time dependent risk models: Ruin problems and computations of ruin functions. Dual queuing models in continuous time and numerical evaluation of ruin functions. (6)

References :

1. Hull, J. C. – Options, Futures and other Derivatives.
2. Ross, S. M. – An Introduction to Mathematical Finance.
3. Neftci, S. N. – An Introduction to Mathematical Financial Derivatives.
4. Elliott, R. J. and Kopp, P. E. – Mathematics of Financial Markets.
5. Merton, R. C. Continuous – Time Finance.
6. Daykin, C. D., Pentikainen, T. and Pesonen, M. – Practical Risk Theory for Actuaries.
7. Rolski, T., Schmidli, H., Schmidt, V. and Teugels, J. – Stochastic Processes for Insurance and Finance.

Optional Subject

Seismology

(Applied Stream)

Marks : 100 (SEE: 80; IA: 20)

Vibrations and Waves : Theory of elastic waves in perfectly elastic media. Vibration and waves. Seismological considerations. Plane waves Standing waves. Dispersion of waves. Energy in plane wave motion. General solution of wave equation. (10)

Bodily elastic waves: P wave (P-Wave) and Secondary wave (S- waves). The effect of gravity fluctuations. Effect of deviation from perfect elasticity. The Jeffereys–Lomnitz Law. (5)

Surface elastic waves: Surface waves along the plane boundary between two homogeneous perfectly elastic media. Rayleigh waves. Love waves. Dispersion curves. Rayleigh waves in presence of a surface layer. Seismic surface waves. (7)

Reflection and refraction of elastic waves: Laws of reflection and refraction. General equations for the two media. Case of incident Surface Horizontal (SH-wave), P-wave and Surface Vertical (SV-wave) incident against free plane boundary. Reflection and refraction of seismic waves. Lamb's problem-line load suddenly applied on elastic half-space. Refraction of dispersed waves. (10)

Seismic rays in a spherically stratified earth model: The parameter p of a seismic ray. Relation between p , Δ , T for a given family of rays. Features of the relations between Δ and T corresponding to certain assigned types of variation with r . Derivation of the P-and S-velocity distributions from the (T , Δ) relations. Special velocity distributions, e.g. curvature of a seismic ray, rays in a homogeneous medium, circular rays. (10)

Amplitude of the surface motion due to seismic waves: Energy per unit area of wave front in an emerging wave. Relation between energy and amplitude Movements of the outer surface arising from an incident wave of given amplitude. Amplitude as a function of Δ . Loss of energy. (10)

Travel-time analysis: Parameters of earthquakes. Epicentral distance and azimuth of an observing station from an epicentre. Theory of the evolution of the main P travel-time table. (7)

Seismology and the earth's upper layers and interior Positions : Theory of travel-times near earthquakes. Physical properties of earth's upper layers. Discontinuities within the earth. (7)

References :

1. Byerly, P. : Seismology.
2. Richter, C. F. : Elementary Seismology

3. Love, A. E. H. : Some Problems of Geodynamics.
4. Bullen, K. E. : An Introduction to the Theory of Seismology.
5. Bath, M. : Theory of Seismology.

Optional Subject
Computational Biology
(Applied Stream)

Marks : 100 (SEE: 80; IA: 20)

A brief review of computational aspects molecular biology.

Basic concepts of molecular biology: DNA and proteins. The central dogma. Gene and Genome sequences. (10)

Restriction maps : Graphs. Interval graphs. Measuring fragment sizes. (10)

Algorithms for double digest problem (DDP) : Algorithms, and complexity Analysis. Mathematical programming approaches to DDP : Integer programming. Partition problems. Travelling Salesman Problem (TSP). Simulated Annealing (SA). (15)

Sequence assembly: Sequencing strategies. Assembly in practices, fragment overlap statistics, fragment alignment, sequence accuracy. (13)

Sequence comparisons methods: Local and global alignment. Dynamic programming solution method. Multiple sequence alignment. (12)

Stochastic Approach to sequence alignment and sequence pattern-Hidden: Markov chain method for biological sequences. (10)

References :

1. Waterman, M. S. : Introduction to Computational Biology.
2. Baxevanis, A. and Ouellette, B. : Bioinformatics-A Practical Guide to the Analysis of Genes and Proteins.
3. Floudas, C. A. : Nonlinear and Mixed -Integer Optimization.
4. Bellman, R. and Krush, R. : Dynamic Programming – Bibliography of Theory and Applications.
5. Bellman, R. and Dreyfus, S. E. : Applied Dynamic Programming.
6. Rao, S. S. : Engineering Optimization.
7. Devis, L. : Genetic Algorithms and Simulated Annealing.

Optional Subject
Mathematical Biology
(Applied Stream)

Marks : 100 (SEE: 80; IA: 20)

Diffusion Model: The general balance law, Fick's law, diffusivity of motile bacteria. (5)

Models for Developmental Pattern Formation: Background, model formulation, spatially homogeneous and inhomogeneous solutions, Turing model, conditions for diffusive stability and instability, pattern generation with single species model. (10)

Deterministic Epidemic Models: Deterministic model of simple epidemic, Infection through vertical and horizontal transmission, General epidemic- Karmac-Mackendric Threshold Theorem, Recurrent epidemics, Seasonal variation in infection rate, allowance of incubation period. Simple model for the spatial spread of an epidemic. Proportional Mixing Rate in Epidemic: Introduction, SIS model with proportional mixing rate, SIRS model with proportional mixing rate. (10)

Stochastic Epidemic Models: Introduction, stochastic simple epidemic model, Yule-Furry model (pure birth process), expectation and variance of infective, calculation of expectation by using moment generating function. (5)

Eco-Epidemiology: Introduction, host-parasite-predator systems, viral infection on phytoplankton zooplankton (prey-predator) system. (5)

Models for Population Genetics: Introduction, basic model for inheritance of genetic characteristic, Hardy-Wienberg law, models for genetic improvement, selection and mutation- steady state solution and stability theory. (5)

Blood flow models: Basic concepts of blood, special characteristics of blood flow. Application of Poiseulle's law to the study of bifurcation in an artery. Pulsatile flow of blood in rigid and elastic tubes. Aortic diastolic-systolic pressure waveforms. Moen-Korteweg expression for pulse wave velocity in elastic tube. Blood flow through artery with mild stenosis. (10)

Models for other fluids: Peristaltic motion in a channel and in a tube. Two dimensional flow in renal tubule. Lubrication of human joints. (5)

References:

1. J.D.Murray : Mathematical Biology, Springer and Verlag.
2. Mark Kot: Elements of Mathematical Ecology, Cambridge Univ. Press.
3. Leach Edelstein-Keshet: Mathematical Models in Biology, Birkhauser Mathematics Series.

4. F. Verhulst: Nonlinear Differential Equations and Dynamical Systems, Springer-Verlag.
5. R. M. May: Stability and Complexity in Model Ecosystem.
6. N.T.J.Bailey: The Mathematical Theory of Infectious Diseases and its Application, 2nd edn. London.
7. H. I. Freedman - Deterministic Mathematical Models in Population Ecology.
8. L.A.Segel (1984): Modelling Dynamical Phenomena in Molecular Biology, Cambridge University Press.
9. Vincenzo Capasso (1993): Lecture Notes in Mathematical Biology (Vol. No. 97)-Mathematical Structures of Epidemic Systems, Springer Verlag.
10. Eric Renshaw (1990): Modelling Biological Populations in Space and Time, Cambridge Univ. Press.
11. Busenberg and Cooke (1993): Vertically Transmitted Diseases- Models and Dynamics, Springer Verlag.
12. Fung, Y.C.: Biomechanics.

Optional Subject
Dynamical Oceanography
(Applied Stream)
Marks : 100 (SEE: 80; IA: 20)

Hydrothermic equations of seawater. Gibbs relation, Gibbs-Duhem relation, heat capacities, Vaisala frequency, Determination of the thermodynamic properties of seawater. (6)

Equations of motion of seawater. Conservation of mass and diffusion of salt. Kinematic free surface condition taking mass exchange into account. Equation of motion of seawater considered a viscous compressible fluid referred to a frame rotating with the earth. Energy transport equation. Thermodynamic energy equation. Entropy transfer equation. The closure problem and relation between thermodynamic fluxes and gradients of t , p , s . Properties and consequences of the adiabatic equations. Ertel's formula, potential vorticity and Rossby principle. Approximation of the basic equations - Boussinesq and linear approximations, \square -approximation, Quasi-geostrophic equations. (18)

Wave motions in the ocean. General properties of plane and nearly plane waves. Linearised small-amplitude waves under gravity in rotating stratified ocean-simple gyroscopic and internal waves, internal gravity waves, plane waves, the energetic of plane waves. Long wave equation for a continuously stratified fluid. Wave reflection and wave trapping by lateral boundaries. Nonlinear surface waves : the Stokes approximation, finite-amplitude wave in shallow water. The solitary wave. (18)

Turbulence : Basic concept. Time-averaged form of the momentum and continuity equations for incompressible flow. Eddy coefficients and their estimations. Elementary examples of the application of eddy coefficients. Salinity tongue in an ocean at rest. (10)

Currents in the ocean. Quasi-static approximation. Geostrophic motion in a stratified ocean. Helland-Hansen formula. Stationary accelerate currents. Steady wind-driven currents in a homogeneous ocean. Wind-drift. Characterization of horizontal and vertical motion. Equation satisfied by the total flow function. Sverdrup's curl relation. Western boundary current. Munk's formula. Sverdrup's study of wind driven current in a baroclinic ocean. Munk's theory of wind-driven ocean circulation. (14)

Tides and storm surges. Statistical theory of tides. Tidal harmonics channel theory of tides. (4)

References :

1. P. H. Leblond and L. A. Mysak : Waves in the Ocean.
2. J. Pedlosky : Geophysical Fluid Dynamics.
3. V. M. Kamenkovch : Fundamentals of Ocean Dynamics.
4. O. M. Philips : Dynamics of the Upper Ocean.

Optional Subject

Applied Functional Analysis

(Applied Stream)

Marks : 100 (SEE: 80; IA: 20)

Review of basic properties of Hilbert spaces. (5)

Convex programming : Support functional of a convex set. Minkowski functional, Separation theorem. Kuhn-Tucker optimality theorem. Mini-Max theorem. Farkas theorem. (10)

Spectral theory of operators : Spectral theory of compact operators. Operators on a separable Hilbert space. Krein factorization theorem for continuous kernels and its consequences. l_2 spaces over Hilbert spaces. Multilinear forms. Analyticity theorem. Nonlinear Volterra operators. (20)

Semigroups of linear operators : General properties of semigroups. Generation of semigroups. Dissipative semigroups. Compact semigroups. Holomorphic semigroups. Elementary examples of semigroups. Extensions. Differential equations. Cauchy problem. Controllability. State reduction. Observability. Stability and stabilizability. Evaluation equations. (20)

Optional control theory : Linear quadratic regulator problems with finite and infinite time intervals. Concept of hard constraints. Final value control. Time optimal control problems. (15)

References :

1. A. V. Balakrishnan : Applied Functional Analysis.
2. N. Dunford and J. T. Schwartz : Linear Operators, Vols. I & II.
3. S. G. Krein : Linear Differential Equations in a Banach Space.
4. K. Yosida : Functional Analysis.
5. M. Avriel : Nonlinear Programming – *Analysis and Methods*.
6. L. Mangasarian : Nonlinear Programming.
7. S. S. Rao : Optimization – *Theory and Applications*.
8. E. Kreyszing : Introductory Functional Analysis with Applications.
9. D. H. Grieffel : Applied Functional Analysis.
10. J. Zabczyk : Mathematical Control Theory – *An Introduction*.
11. W. L. Brogan : Modern Control Theory.
12. H. Kwakernaak and R. Sivan : Linear Optimal Control Systems.
13. A. Isidori : Nonlinear Control Systems.
14. S. G. Tzafestas : Methods and Applications of Intelligent Control.

Optional Subject**Advanced Numerical Analysis (Theory and Practical)****(Applied Stream)****Marks : 100****Advanced Numerical Analysis: Theory (SEE: 50; IA: 12)**

Interpolation : Newton's bivariate interpolation Triangular interpolation, Bilinear interpolation.

Approximation: Rational approximation, Continued fraction approximation, Pade approximation.

Solution of polynomial equation : Birge-Vieta method, Bairstaw method. (8)

Solution of linear system of equations : Direct methods : Cholosky method, Partition method, error estimations. Iterative methods : Different iterative schemes, Optimal relaxation parameter for SOR method, Convergence analysis. (16)

Eigen value problems of real symmetric matrices : Bounds of Eigenvalues, Householder's method, Given's method, Inverse power method. (10)

Solution of nonlinear system of equations : Newton's method, Steepest- Descent method, Convergence analysis. (6)

Numerical solution of boundary value problems of Ordinary differential equations : Finite-difference method, Newton-Raphson method (second order equation), error estimations. (5)

Numerical solution of partial differential equations : Introduction to Elliptic, Parabolic and Hyperbolic equations. Explicit methods : Schmidt method, Dufort-Frankel method, Convergence and stability analysis. Implicit methods : Crank-Nicolson method, convergence and stability analysis, Matrix method. (6)

Numerical solution of integral equations : Finite - difference method, Cubic spline method, Method using Generalized quadrature. (5)

Finite Element Methods : Introduction to Finite Element methods. Weighted residual methods : Least square method, Partition method, Variational method : Ritz method. (5)

Finite elements : Line segment element, Triangular element, Rectangular element, Curved-boundary element. (5)

Finite element methods: Ritz finite element method, Least square finite element method, Convergence, Completeness and Compatibility analysis. Boundary value problems in ordinary differential equations : Mixed boundary conditions - Galerkin method. (10)

References :

1. E. V. Krishnamurthy and S. K. Sen : Numerical Algorithms Computations in Science and Engineering.
2. Hildebrand, F. B. : An Introduction to Numerical Analysis.
3. Atkinson, K. E.: An Introduction to Numerical Analysis.
4. Collatz, L. : Functional Analysis and Numerical Mathematics.
5. Fox, L. : Numerical Solution of Ordinary and Partial Differential Equations.
6. Ames, W. F. : Numerical Methods of Partial Differential Equations.
7. Strang, G., Fix, G. : An Analysis of the Finite Element Methods.
8. Zienkiewicz, O. C. : The Finite Element Methods in Structural and Continuum Mechanics.
9. Jain, M. K., Iyengar, S. R. K., Jain, R. K. : Numerical Methods for Scientific and Engineering Computations.
10. Jain, M. K. : Numerical Solution of Differential Equations.
11. Baker, C. T. H. and Phillips, C. : The Numerical Solution of Non-linear Problems.

Advanced Numerical Analysis: Practical (SEE: 30*; IA: 08)

(*Laboratory Assignment = 5 marks + Viva- Voce = 5 marks

+ Compilation and Execution of Two Problems = 20 marks).

1. Newton's method for finding real roots of simultaneous equations.
2. Graeffee's Root-squaring method (up to biquadratic).
3. Bairstow's method (up to biquadratic).
4. Q-D (Quotient-Difference) method.
5. Matrix inversion : Cholesky method.
6. Eigen value problems : Jacobi's method, Inverse Power method.
7. Numerical Solution of ODEs : Explicit and implicit R-K (Runge-Kutta) methods, Predictor-Corrector methods, Adams' method.
8. Boundary value problems : Finite- difference method.
9. Numerical solutions of PDEs : Crank – Nicolson method.
10. Cubic Spline interpolation using the General Form.
11. Integral equation : Monte – Carlo method.

Practical Examination Related Criteria :

- (i) Laboratory clearance be taken by the students prior to commencement of practical examination.
- (ii) The Lab. Assignment Dissertations of the students be submitted prior to commencement of practical examination.
- (iii) Duration of practical examination will be 4 (Four) hours.
- (iv) One external examiner be appointed for practical examination.

References :

1. Krishnamurthy, E. V. and S. K. Sen : Numerical Algorithms Computations in Science and Engineering.
 2. Balaguruswamy, E. : Programming in ANSI C.
 3. Xavier, C : C and Numerical Methods.
-

Optional Subject
Compressible Fluid Dynamics
(Applied Stream)
Marks: 100 (SEE: 80; IA: 20)

Compressible Fluid: Compressibility of Fluids, System and Control Volume, Thermodynamic Process and Cycle, Laws of Thermodynamics, Stored Energy and Energy in Transition, Entropy, Isothermal-Adiabatic and Isentropic process, Perfect gas. (6)

Conservation Laws for Compressible Fluids: Extensive and intensive properties, Conservation of mass and Continuity equation, Conservation of Momentum and Momentum equation, Conservation of Energy and Energy equation. (7)

Basic Concepts of Compressible Flow: Velocity of Sound, Mach Number, Subsonic and Supersonic Flow, Stagnation Condition, Relation between Stagnation and Static Properties, Kinetic form of Steady Flow Energy Equation, Critical Speed of Sound, Stream Thrust and Impulse Function. (10)

Isentropic Flow: Governing equations, Effect of Area Variation, Nozzle, Diffuser, Choking, Isentropic Flow Relations, Differential Equations in terms of Area variation and Solution. (6)

Normal Shock Waves: Compression Wave and Expansion Wave, Governing Equations for Normal Shock Waves, Hugoniot Curve, Prandtl-Meyer Equation, Mach Number Downstream of Normal Shock, Property Ratios across Normal Shock, Stagnation to Static Pressure Ratios, Change in Entropy across Normal Shock, Rankine-Hugoniot Relations. (12)

Oblique Shock Waves: Compression Shock Wave and Expansion Fan, Upstream and Downstream Velocity Triangles, Oblique Shock Relations, Deflection and Wave Angle, Prandtl Velocity Equation for Oblique Shock Wave, Mach Lines, Prandtl-Meyer Flow, Prandtl-Meyer Angle. (12)

Rocket Propulsion: Rocket Propulsion Parameters, Effective Jet Velocity, Characteristic Velocity, Exit Velocity of Jet, Design Parameters for Rocket Engine, Propellants, Combustion, Rocket Equation, Altitude Gain during Vertical Flight, Escape Velocity. (7)

References:

1. Thompson, P. A., Compressible Fluid Dynamics.
2. Shaprow, A. H., Compressible Fluid Flow.

3. Niyogi, P., Inviscid Gas Dynamics.
4. Oswatitsch, K., Gas Dynamics.
5. Yahya, S. M., Fundamentals of Compressible Flow.

Optional Subjects for Only Pure Stream

Optional Subject

Advanced Real Analysis

(Pure Stream)

Marks : 100 (SEE: 80; IA: 20)

Representation of real numbers by series of radix fractions. Sets of real numbers, Derivatives of a set. Points of condensation of a set. Structure of a bounded closed set. Perfect sets. Perfect kernel of a closed set. Cantor's nondense perfect set. Sets of first and second categories, residual sets. (12)

Baire one functions and their basic properties. One-sided upper and lower limits of a function. Semicontinuous functions. Dini derivatives of a function. Zygmund's monotonicity criterion. (12)

Vitali's covering theorem. Differentiability of monotone functions and of functions of bounded variation. Absolutely continuous functions, Lusin's condition (N), characterization of AC functions in terms of VB functions and Lusin's condition. (6)

Concepts of VB^* , AC^* , VBG^* , ACG^* etc. functions. Characterization of indefinite Lebesgue integral as an absolutely continuous function. (6)

Generalized Integrals : Gauge function. Cousin's lemma. Role of gauge function in elementary real analysis. Definition of the Henstock integral and its fundamental properties. Reconstruction of primitive function. Cauchy criterion for Henstock integrability. Saks-Henstock Lemma. The Absolute Henstock Integral. The McShane integral. Equivalence of the McShane integral, the absolute Henstock integral and the Lebesgue integral. Monotone and Dominated convergence theorems. The Controlled convergence theorem. (16)

Definition and elementary properties of the Perron integral and its equivalence with the Henstock integral. (6)

Definition of the (special) Denjoy integral and its equivalence with the Henstock integral (characterization of indefinite Henstock integral as a continuous ACG* function). (4)

Density of arbitrary sets. Approximate continuity. Approximate derivative. (4)

References :

1. E. W. Hobson : The Theory of Functions of a Real Variable (Vol. I and II).
2. I. P. Natanson : Theory of Functions of a Real Variable (Vol. I and II).
3. R. A. Gordon : The Integrals of Lebesgue, Denjoy, Perron and Henstock, Amer. Math. Soc. Graduate Studies in Math., Vol. 4, 1994.
4. W. F. Pfeffer : The Riemann Approach to Integration - Local Geometric Theory.
5. R. Henstock : Lectures on the Theory of Integration.
6. P. Y Lee : Lanzhou Lectures on Henstock Integration.
7. S. Schwabik : Generalized Ordinary Differential Equations.
8. E. J. McShane : Unified Integration.
9. S. Saks : Theory of the Integral.

Optional Subject

Advanced Partial Differential Equations

(Pure Stream)

Marks : 100 (SEE: 80; IA: 20)

Common linear PDEs and their boundary conditions Cauchy data and the Cauchy-Kowalewski expansion, Weak solutions of linear PDEs Well-posedness , Classification of PDEs and PDE systems from their principal symbol , (5)

Parabolic PDE, Heat Equation in \mathbb{R}^n , Fundamental solution, Heat ball, Maximum Principle. (5)

Hyperbolic PDE in \mathbb{R}^n , Scalar conservation laws and the Riemann problem Generalized functions and the delta-function (10)

Hamilton Jacobi Equation, Elliptic PDE, Green's functions for ODEs Green's functions for applications in Laplace, Poisson and Helmholtz equations , Review of harmonic functions, Extension of maximum principles (15)

Variational problems. Euler Lagrange equations,(10)

Introduction to sobolev space.(5)

Solution of PDE by Finite element method Using MATLAB.(10)

References.

Barros-Neto- An introduction to the Introduction to the Theory of Distributions

Adams- Sobolev Spaces,

Kesavan - Topics in Functional Analysis and Applications

Evans - Partial Differential Equations

H.Brezis-Analyse Fonctionnelle

Optional Subject

Advanced Complex Analysis – I

(Pure Stream)

Marks : 100 (SEE: 80; IA: 20)

The functions- $M(r)$ and $A(r)$. Hadamard theorem on the growth of $\log M(r)$, Schwarz inequality, Borel-Caratheodory inequality, Open mapping theorem. (10)

Dirichlet series, abscissa of convergence and abscissa of absolute convergence, their representations in terms of the coefficients of the Dirichlet series. The Riemann Zeta function, the product development and the zeros of the zeta functions. (10)

Entire functions, growth of an entire function, order and type and their representations in terms of the Taylor coefficients, distribution of zeros. Schottky's theorem (no proof). Picard's first theorem. Weierstrass factor theorem, the exponent of convergence of zeros. Hadamard's factorization theorem, Canonical product, Borel's first theorem. Borel's second theorem (statement only). (16)

Multiple-valued functions, Riemann surface for the functions \sqrt{z} , $\log z$ (3)

Analytic continuation, uniqueness, continuation by the method of power series, natural boundary, existence of singularity on the circle of convergence. Functions element, germ and complete analytic functions. Monodormy theorem. (12)

Conformal transformations, Riemann's theorems for circle, Schwarz principle of symmetry, Schwarz-Christoffel formula (statement only) with applications.

Univalent functions, general theorems, sequence of univalent functions, sufficient conditions for univalence. (8)

References :

1. E. T. Copson : An Introduction to the Theory of Functions of a Complex Variable.
2. E. C. Titchmarsh : The Theory of Functions.
3. A. I. Markushevich : Theory of Functions of a Complex Variable (Vol. I, II & III).
4. L. V. Ahlfors : Complex Analysis.
5. J. B. Conway : Functions of One Complex Variable.
6. A. I. Markushevich : The Theory of Analytic Functions, A Brief Course.
7. G. Valiron : Integral Functions.
8. C. Caratheodory : Theory of Functions of a Complex Variable.
9. R. P. Boas : Entire Functions.

Optional Subject

Advanced Complex Analysis – II

(Pure Stream)

Marks : 100 (SEE: 80; IA: 20)

Harmonic functions, Characterisation of Harmonic functions by mean-value property. Poisson's integral formula. Dirichlet problem for a disc. (12)

Doubly periodic functions. Weierstrass Elliptic function. (16)

Entire functions, $M(r, f)$ and its properties (statements only). Meromorphic functions. Expansions. Definition of the functions $m(r, a)$, $N(r, a)$ and $T(r, f)$.

Nevanlinna's first fundamental theorem. Cartan's identity and convexity theorems. Orders of growth. Order of a meromorphic function. Comparative growth of $\log M(r)$ and $T(r)$. Nevanlinna's second fundamental theorem. Estimation of $S(r)$ (Statement only). Nevanlinna's theorem on deficient functions. Nevanlinna's five-point uniqueness theorem. Milloux theorem. (25)

Functions of several complex variables. Power series in several complex variables. Region of convergence of power series. Associated radii of convergence. Analytic functions. Cauchy-Riemann equations. Cauchy's integral formula. Taylor's expansion. Cauchy's inequalities. Zeros and Singularities of analytic functions. Weierstrass preparation theorem (statement only). (15)

References :

1. E. C. Tittmarsh, The Theory of Functions.
 2. E. T. Copson, An Introduction to the Theory of Functions of a Complex Variable.
 3. A. I. Markushevich, Theory of Functions of a Complex Variable, (Vol. I, II, III).
 4. W. Kaplan, An Introduction to Analytic Functions.
 5. H. Cartan, Theory of Analytic Functions.
 6. W. K. Hayman, Meromorphic Functions.
 7. L. Yang, Value Distribution Theory.
 8. R. C. Gunning and H. Rossi, Analytic Functions of Several Complex Variables.
 9. B. A. Fuks. An Introduction to the Theory of Analytic Functions of Several Complex Variables.
 10. Bochner and Martin. Several Complex Variables.
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Optional Subject Advanced Functional Analysis

(Pure Stream)

Marks : 100 (SEE: 80; IA: 20)

Complete orthonormal sequence and separability in Hilbert spaces. Complete orthonormal sequence in $L[0, 2\pi]$, Isometric isomorphism of every infinite dimensional separable Hilbert space with the space l_2 . (6)

The weak* topology of the conjugate of a normal space, Banach-Alaoglu theorem. Annihilators of subspaces of X and X^* , where X is a Banach space ; Conjugates of subspaces and of quotient spaces of X . The Krein-Milman theorem on extreme points in normed spaces. (6)

Representation theorems for bounded linear functionals on $C[a, b]$ and on ℓ_p ($1 \leq p < +\infty$). (4)

Stone-Weierstrass theorem, Approximation in normed spaces, Best approximation, and uniqueness. (4)

Reflexive spaces, Reflexivity of Hilbert spaces, Reflexive Banach space, Subspaces of reflexive spaces, Bounded sequence contains a weakly convergent subsequence, Existence of an element of smallest norm, Strict convexity, Uniform convexity in relation to reflexivity. (12)

Spectrum of a bounded linear operator, Spectral mapping theorem, Spectrums of completely continuous operator and of self adjoint operator. Spectral representation of self-adjoint operator. (8)

Banach Algebra, Banach Algebra with identity. Resolvent operator, Gelfand transform in commutative Banach Algebra. (10)

Gateaux derivative, uniqueness, representation when domain and range are finite dimensional. Frechet derivative, relation with Gateaux derivative, and complete continuity of Frechet derivative. (10)

References :

1. G. Bachman and L. Narici : Functional Analysis.
2. S. Berberian : Introduction to Hilbert Spaces.
3. A. L. Brown and A. Page : Elements of Functional Analysis.

4. J. B. Conway : A Course in Functional Analysis.
5. N. Dunford and L. Schwartz : Linear Operators.
6. E. Kreyszig : Introductory Functional Analysis with Applications.
7. B. V. Limaye : Functional Analysis.
8. W. Rudin : Functional Analysis.
9. B. K. Lahiri : Elements of Functional Analysis.
10. E. Rickart : Banach Algebra.

Optional Subject
Set-Valued Analysis
(Pure Stream)

Marks : 100 (SEE: 80; IA: 20)

Upper limit and Lower limit, and Limit of sequences of sets in metric spaces ; Basic properties and examples. Calculus of upper and lower limits. Zorankiewicz's compactness theorem in separable metric spaces. (10)

Set-valued maps (Multifunctions) ; their graph, domain, image, inverse. Inverse image and Core of a set by a set-valued map. The operations of union, intersection, difference, vector sum (in vector spaces), composition product and square product of set-valued maps ; their basic properties. (14)

Upper semicontinuous, Lower semicontinuous, continuous and Lipschitz set-valued maps in metric spaces and normed spaces. Upper and lower limits of set-valued maps. Proper set-valued maps. Marginal function. Maximum theorem. Ekeland's Variational Principle theorem. Michael's theorem on continuous selection. (15)

Closed convex processes in normed spaces ; Open mapping theorem, Closed graph theorem, Uniform boundedness theorem. (5)

Contingent cone, Adjacent cone and Circatangent cone to subsets of normed spaces ; their basic properties. Special properties of the tangent cones to convex sets. Contingent derivative. Adjacent derivative and circatangent derivative of set-valued maps in normed spaces ; Their basic properties and expressions as limits of differential quotients. (16)

References :

1. J. P. Aubin and H. Frankowska : Set-valued Analysis.
 2. K. Kuratowski : Topologie.
 3. H. Frankowska : Set-Valued Analysis and Control Theory.
 4. J. P. Audin and I. Ekeland : Applied Non-linear Analysis.
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Optional Subject**Abstract Harmonic Analysis****(Pure Stream)****Marks : 100 (SEE: 80; IA: 20)**

Banach Algebra : Banach Algebras, basic concepts, Gelfand theory, The spectral Theorem, Spectral theory of *-representations.

Locally compact groups : Harr measure, Unimodular group, Homogeneous spaces.

Representation Theory : Unitary representation, Representation of a group and its group algebra, Functions of positive type.

Analysis on Locally compact groups : Dual group, Fourier transform, Potriagin duality.

Analysis on Compact groups : Representation of Compact groups, The Peter-Weyl Theorem.

References :

1. A Course in Abstract Harmonic Analysis, G. B. Folland
 2. E. Hewitt and K. Ross : Abstract Harmonic Analysis, (Vol.1).
 3. L. Loomis : An Introduction to Abstract Harmonic Analysis.
 4. W. Rudin : Fourier Analysis on Groups.
 5. G. Bachman : Elements of Abstract Harmonic Analysis.
 6. W. Rudin : Real and Complex Analysis.
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Optional Subject

Advanced General Topology

(Pure Stream)

Marks : 100 (SEE: 80; IA: 20)

Locally Connected space, Various Disconnected spaces, and Quotient Spaces : Local Connected spaces, Zero-dimensional spaces, totally and extremally disconnected spaces, characterizations and their basic properties. Quotient spaces. (6)

Nets and Filters: Inadequacy of sequence, Directed set, definition of net, convergence by net. Cluster point of a net, subnet, ultranet, Topological concepts via nets. (5)

Definition of a filter. Free and fixed filter. Filter bases, image and inverse image of filter base and filter, induced filter. Ultrafilter and its existence and characterization. Convergence of filters. Properties of convergence of filters. Cluster point of a filter and its properties. Characterizations of compactness in terms of nets and filters. Alternative proof of Tychonoff product Theorem using ultranet / ultrafilter. Net based on filter, filter generated by net. (10)

Compactification : Locally compact spaces : Examples and various characterizations, compactification of topological spaces. Alexandroff compactification. Stone-Cech compactification. Cardinality of $\beta\mathbb{N}$. (10)

Paracompactness : Star refinement, barycentric refinement and their relation. Various characterizations of paracompactness. A. H. Stone's theorem concerning paracompactness of metric spaces. Interconnection between paracompactness and (i) Hausdorffness, (ii) Regularity and (iii) Lindelöfness. Properties of paracompactness with regard to subspaces and product space. (10)

Embedding and Metrization : Evaluation map, Embedding theorem for Tychonoff spaces, Urysohn's metrization theorem. (5)

Uniform spaces : Definition and examples of uniform spaces. Base and subbase of a uniformity, uniform topology. Uniformity and separation axioms. Uniformizable spaces. Uniform continuity and product uniformity. Uniform property. Uniformity of pseudometric spaces and uniformity generated by a family of pseudometric. Compactness of uniform spaces. Cauchy filter. Relation between completeness and compactness in uniform spaces. (10)

Proximity spaces : Definition and examples. Topology induced by proximity. Alternate description of proximity (the concept of δ -neighbourhood). Separated proximities. Proximal

neighbourhoods. p -map, p -isomorphism. Subspaces and product of proximity spaces. Proximities induced by uniformities. Compactness and proximities. (5)

$C(X)$ and $C^*(X)$: The function rings $C(X)$ and $C^*(X)$, C -embedded and C^* embedded sets in X . Urysohn's extension theorem, Z -filters and Z -ultrafilters on X , their duality with ideals and maximal ideals of $C(X)$. Fixed ideals and compact spaces. (9)

References :

1. J. L. Kelley : General Topology.
2. S. Willard : General Topology.
3. J. Dugundji, Topology.
4. R. Engelking : Outline of General Topology.
5. S. A. Naimpally and B. D. Warrack : Proximity Space.
6. J. Nagata : Modern General Topology.
7. L. Gillman and M. Jerison : Rings of continuous functions.
8. J. Nagata : Modern Dimension Theory.

Optional Subject

Advanced Algebraic Topology

(Pure Stream)

Marks : 100 (SEE: 80; IA: 20)

Covering spaces : Basic properties, Classification of covering spaces. Universal covering spaces. Applications – Borsuk Ulam Theorem. (12)

Higher Homotopy Groups : Basic properties and examples. Homotopy Groups of Spheres. Relation between homology groups and homotopy groups. Lefschetz fixed point theorem. Brouwer fixed point theorem. (15)

Singular Homology Theory : Singular Chain Complex. Singular Homology group. Chain map, induced map between homology groups. Chain homotopy, Mayer-Vietoris sequences. Axioms for homology theorem. (15)

Cohomology and Duality Theorems : Definitions and Calculation Theorems. Poincaré duality. Alexander duality and Lefschetz duality. (8)

CW-complexes : Definition, Cellular maps. Homotopy groups of CW-complexes. Whitehead Theorem. Homology theory of CW-complexes. Betti number and Euler characteristics. Excision theorem and cellular homology, Hurewicz theorem. Fiber spaces. Presheaves. Fine presheaves. Application of cohomology to presheaves. (15)

References :

1. Fred. H. Croom : Basic Concepts of Algebraic Topology.
2. C. R. F. Maunder : Algebraic Topology.
3. Edwin H. Spanier : Algebraic Topology.
4. J. Mayer : Algebraic Topology.
5. B. Gray : Homotopy Theory.
6. J. Dugundji : Topology.
7. Allen Hatcher : Algebraic Topology.

Optional Subject

Optional Subject Advanced Algebra –I (Pure Stream)

Marks : 100 (SEE: 80; IA : 20)

Ideals and Bi-ideals: Definitions, related concepts of semi groups and rings, their different types of classifications and generalizations-relevant results, Fuzzy and Anti fuzzy algebraic treatment of them. (20)

Finite field and field extensions: Definitions and study of important properties-related results and their verifications with examples. (20)

Geometric Constructions: Constructible Real Numbers, Trisection of 60° Angle and square the circle by straight edge and compass. Duplication of a cube. Construction of a Regular Septagon. Constructibility of Regular 9-gon and regular 20-gon. (10)

Advanced module theory: Review of different kinds of modules-some advanced theories, products and co-products, injective modules, tensor products, modules over a principal ideal domains, finitely generated abelian groups. (10)

Semi groups: Review of earlier related concepts, idea of regular semi groups, completely regular semi groups, intra regular semi groups etc and their related properties, semi lattices of groups. (10)

References :

1. A. G. Kurosh : The Theory of Groups.
2. T. W. Hungerford : Algebra
3. S. T. Hu : Elements of Modern Algebra.
4. E. Artin : Galois Theory (2nd Edition).
5. N. Jacobson : Lectures in Abstract Algebra (Vol. -I).
6. M. Nagata : Field Theory.
7. J. A. Gallian : Contemporary Abstract Algebra.
8. D. S. Malik, J. M. Mardeson and M. K. Sen : Fundamental of Abstract Algebra.
9. M. R. Adhikari and Avishek Adhikari : Groups, Rings, and Modules with Applications.
10. J. N. Mordeson, D. S. Malik and N. Kuroki : Fuzzy Semigroups.

Optional Subject**Advanced Algebra – II****(Pure Stream)****Marks : 100 (SEE: 80; IA: 20)**

Modules : Modules Homomorphisms. Exact sequences. Free modules, Projective and injective modules. Divisible abelian groups. Embedding of a module in an injective module. (6)

Modules over PID, Torsion-free modules, Finitely generated modules over PID. (4)

Tensor product of modules, Tensor product of free modules. (5)

Commutative Rings and Modules : Noetherian and Artinian modules, Composition series in modules. Primary decomposition of a submodule of a module. (7)

Noetherian rings, Cohen's theorem, Krull intersection theorem, Nakayama lemma. Hilbert basis theorem. (5)

Extension of a ring, Integral extension of a ring, Integral closure, Lying-over and Going-up theorems. (3)

Transcendence base of a field over a subfield. Algebraically independence subset of an extension field over a field. Algebraically closed field extensions of isomorphic fields with equal transcendence degree are isomorphic. (5)

Affine varieties of algebraic sets. Noether normalization lemma, Hilbert Nullstellensatz. (5)

Structure of Rings : Left artinian rings, Simple rings, Primitive rings, Jacobson density theorem, Wedderburn Artin theorem on simple (left), Artinian rings. (5)

The Jacobson radical, Jacobson semisimple rings, subdirect product of rings, Jacobson semisimple rings as subdirect products of primitive rings, Wedderburn-Artin theorem on Jacobson semisimple (left), Artinian rings. (6)

Simple and Semisimple modules, Semisimple rings, Equivalence of semisimple rings with Jacobson (left) Artinian rings, Properties of semisimple rings, Characterizations of semisimple rings in terms of modules. (4)

Group Representations: Group rings, Maschke's theorem, Character of a representation, Regular representations, Orthogonality relations, Burnside's $p^a q^h$ theorem. (10)

References :

11. Serge Lang : Algebra.
12. Nathan Jacobson : Basic Algebra (Vol. II).
13. M. Atiyah and I. G. MacDonald : Introduction to Commutative Algebra.
14. O. Zarisky and P. Samuel : Commutative Algebra (Vols. I and II).
15. D. S. Malik, John M. Mordeson, and M. K. Sen : Fundamentals of Abstract Algebra.
16. N. McCoy : Theory of Rings.
17. I. N. Herstein : Non-Commutative Rings.
18. T. Y. Lam : A First Course in Non-commutative Rings.
19. C. W. Curtis and I. Reiner : Representation Theory of Finite Groups and Associated Algebras.

Optional Subject

Advanced Geometry – I

(Pure Stream)

Marks : 100 (SEE: 80; IA: 20)

Manifold Theory: Differentiable manifold, Differentiable mapping, Differentiable transformations, Diffeomorphism, Vector field, Integral curve of a vector field, Lie bracket, Immersion, Imbedding, rank of a mapping, f-related vector fields. Total differential of a function. Lie groups. (20)

Geodesics, Convex neighbourhood, Geodesic flow, Minimizing properties of geodesics, convex neighbourhood. (5)

Riemannian Manifolds : Affine connections, Riemannian connections, semi symmetric connections, fibre bundle Basic definitions, Curvature tensor, Ricci tensor, Scalar curvature, Sectional curvature, Properties of Riemann curvature tensor, Bianchi's identities, Conformal curvature tensor, Projective curvature tensor, Jacobi equations Local Isometrics, Lie Derivatives and their elementary properties. (25)

Isometric immersions: The second fundamental form, The fundamental equations, Complete manifolds, Hopf Rinow Theorem, The Theorem of Hadamard. (5)

Spaces of constant curvature, Theorem of Cartan, Hyperbolic spaces, Formulas for the first and second variation of energy. (5)

References :

1. N. J. Hicks : Notes on Differential Geometry.
2. Riemannian Geometry, M. P. Do carmo.
3. A course in Differential Geometry and Lie Groups, S. Kumaresan.
4. S. Kobayasi and K. Nomizu : Foundations of Differential Geometry (Vol. 1).
5. W. M. Boothby : An Introduction to Differentiable Manifold and Riemannian Geometry.
6. Barrett O'Neil : Riemannian Geometry.
7. Barrett O'Neil : Semi-Riemannian Geometry with Application to Relativity.
8. U. C. De and A. A. Sheikh : Geometry of Differentiable Manifolds.

Optional Subject

Advanced Geometry – II

(Pure Stream)

Marks : 100 (SEE: 80; IA: 20)

Complex Manifolds : Almost complex manifolds. Neijenhuis tensor. Complex manifolds. Contravariant almost analytic vector. Almost Hermite manifolds. Linear connection in an almost Hermite manifold. Kähler manifold. Almost Tachibana manifold. Tachibana manifold. Holomorphic sectional curvature. Almost product and almost decomposable manifold. Almost Einstein manifold. (20)

Contact Manifolds : Definition and examples of contact manifolds. Almost contact manifolds. K-contact and Sasakian structures. Sasakian space forms. Nearly Sasakian structures. (15)

Geometry of Theory of Relativity : Introduction to Special and General theory of relativity. Manifolds of special and general theory of relativity. Special theory of relativistic mechanics and electro-dynamics. Metric in a gravitational field. Motion of a free particle in a gravitational field. Einstein law of gravitation. Metrics with spherical, planetary orbits. Symmetries, Killing equations. Curvature collineation, Projective collineation, Conformal collineation (including affine collineation, conformal motion and homothetic motion). (30)

References :

1. K. Yano : Differential Geometry on Complex and Almost Spaces.
2. R. S. Mishra : Structures on a Differentiable Manifold and their Application.
3. D. E. Balair : Contact Manifolds in Riemannian Geometry, Lecture Notes in Math, 509.
4. D. F. Lawden : An Introduction to Tensor Calculus, Relativity and Cosmology.
5. R. Resnik : Introduction to Special Relativity.
6. S. K. Bose : An Introduction to General Relativity.
7. A. N. Matveev : Mechanics and Theory of Relativity.
9. E. Lord : An Introduction to Tensor Calculus. Relativity and Cosmology.

Optional Subject

Ergodic Theory and Topological Dynamics

(Pure Stream)

Marks : 100 (SEE: 80; IA: 20)

Measure Preserving Transformation : Definition and Examples, Recurrence, Ergodicity.

The Ergodic Theorem : Von Neumann's L^2 -ergodic Theorem, Birkhoff's Ergodic Theorem, Disintegrating a measure space over a factor algebra.

Mixing Properties : Poincare Recurrence, Ergodicity of a mixing property, Weakly Mixing, A little spectral theory, Weakly mixing and eigenfunctions, Mixing.

Entropy : Partitions and Subalgebras, Entropy of a Partition, Conditional Entropy, Entropy of a measure preserving transformation, Properties of $h(T, A)$, $h(T)$, some methods for calculating $h(T)$, How Good an Invariant is Entropy, Bernoulli Automorphisms and Kolmogorov Automorphisms, The Pinsker σ -Algebra of a Measure Preserving Transformation, Sequence Entropy.

Topological Dynamics : Recurrent points, Uniform Recurrence and Minimal Systems, Multiple Birkhoff recurrence Theorem and its applications

References:

1. H. Furstenberg, Recurrence in ergodic Theory and combinatorial applications
 - D. J. Rudolph, Fundamentals of Measurable dynamics.
 3. Peter Walters, An introduction to ergodic theory.
 4. M. Einsiedlert and Tomas Ward, Ergodic Theory with a view towards number theory
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(S. Pal)

The Head of the Department of Mathematics